3D video games

3D Game Physics

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Course Plan

lec. 1: Introduction
lec. 2: Mathematics for 3D Games
lec. 3: Scene Graph
lec. 4: Game 3D Physics +
lec. 5: Game Particle Systems
lec. 6: Game 3D Models
lec. 7: Game Textures
lec. 8: Game 3D Animations
lec. 9: Game 3D Audio
lec. 10: Networking for 3D Games
lec. 11: Artificial Intelligence for 3D Games
lec. 12: Game 3D Rendering Techniques

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Animation in games

but, a note on terminology:
in some contexts, procedural means
"produced by a *simple* procedure"
as opposed to "physically simulated"

Non procedural	**Procedural**
Assets!	Physics engine
Fully controlled by artist/designer (dramatic effects!)	Less control
Realism: depends on artist's skill	Physics-driven realism
Does not adapt to context	Auto adaptation to context
Repetition artefacts	Naturally repretition free

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Physics simulation in videogames

- 3D, or 2D
- "soft" real-time
- efficiency
  - 1 frame = 33 msec (at 30 FpS)
  - physics = 5% - 30% max of computation time
- plausibility
  - but not necessarily *accuracy*
- robustness
  - should almost never "explode"
  - it's tolerable to have inconsistency in a few frames, as long as it recovers in subsequent ones

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Physics in games: cosmetics or gameplay?

- Just a graphic accessory? (for realism!)
  - e.g.:
    - particle effects (w/o feedback)
    - secondary animations
    - Ragdolling
- Or a gameplay component?
  - e.g. physics based puzzles
  - Popular approach in 2D (since always!)

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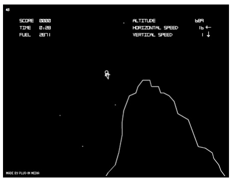

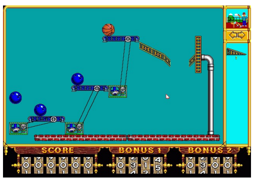

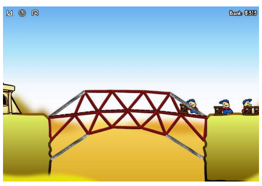

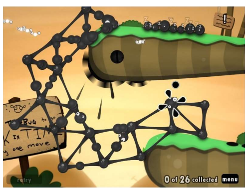

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Physics in games: cosmetics or gameplay?

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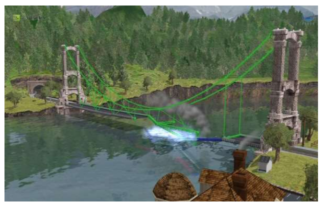
  - e.g.:
    - particle effects (w/o feedback)
    - secondary animations
    - Ragdolling
- Or a gameplay component?
  - e.g. physics based puzzles
  - Rising trend in 3D



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Physics engine: intro

- Game engine module
  - executed in real time at game run-time
- A high-demanding computation
  - on a very limited time budget!
- ...but highly parallelizable
  - potentially, highly parallel

==> good fit for hardware support

(just like the Rendering Engine)

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Hardware for Physics engine

To exploit a strong parallelism, you need a strongly parallel hardware!

- For a brief moment ~2006: PPU
  - "Physics Processing Unit"
  - HW unit specialized for physics
- After that: GP-GPU
  - "General Purpose Graphics Processing Unit"
    = Use of the graphics card for generic tasks
    (not related with 3D computer graphics)
  - or, Cuda (nVidia), OpenCL (openSource)

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Main Software (libraries, SDK)

havok	mostly CPU (Microsoft)
PhysX	CPU+GPU (CUDA) NVidia
Bullet Physics Library	open source, free, HW accelerated (OpenCL) + CPU
Open Dynamics Engine	open source, free
Box2D	2D, open source, free

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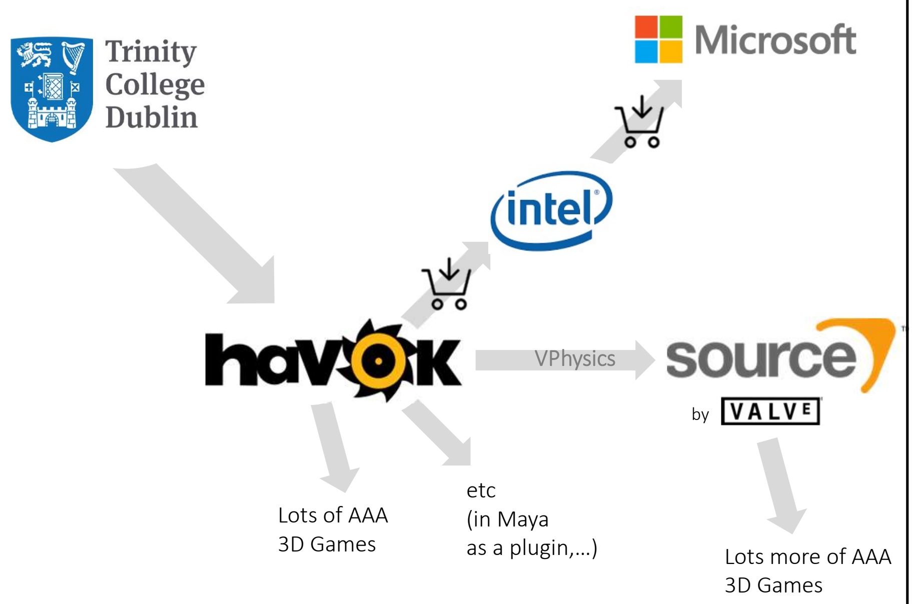
Brief history



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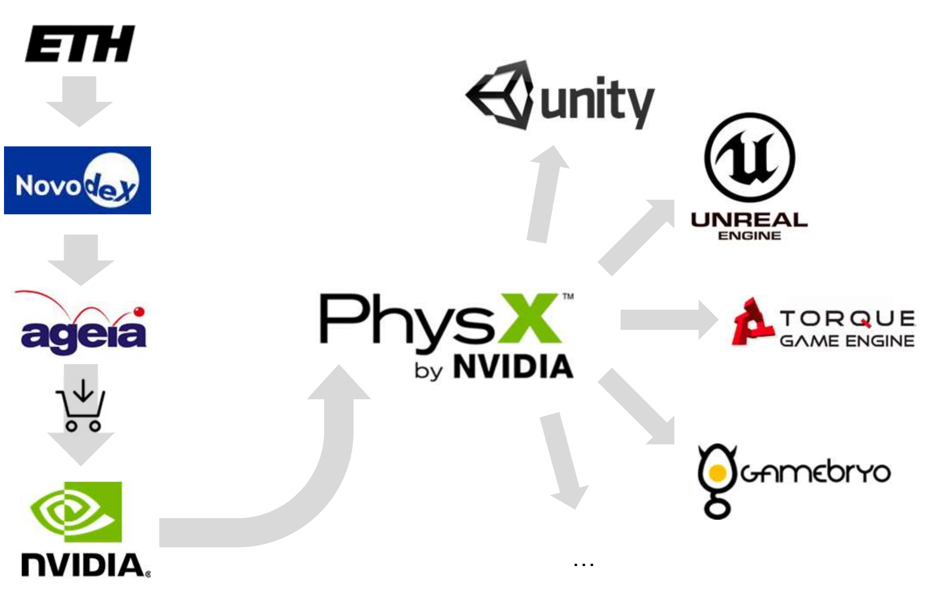
Brief history



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Fields of study

- Dynamics
  - The motion, as a result of forces
  - *"Subject to gravity, how will this pendulum swing?"*
- Statics
  - Equilibrium states, energy minimization states
  - *"In which state(s) can this pendulum be still?"*
- Kinematics
  - The motion itself, irrespective of why it's moving
  - *"If the angular speed of the pendulum is currently X, how fast is the tip moving?"* (or vice versa)

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The 2 tasks of the Physics engine

1. Dynamics (Newtonian)
for objects such as:
   - Particles
   - Rigid bodies
   - Articulated bodies
     - E.g. "ragdolling"
   - Soft bodies
     - Ropes (specific solutions)
     - Cloth (specific solutions)
     - Hair (specific solutions)
     - Free-form deformation bodies (general)
   - Fluids
     - Expensive!

2. Collision handling
   - Collision detection
   - Collision response

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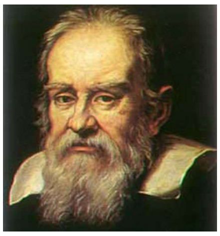
Newtonian Dynamics

- The one with:
  - Masses
  - position and its derivative: velocity
    - and momentum
  - direction and angular velocity
    - and angular momentum
  - forces acceleration…



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Reminder: Spatial placement of a (rigid) object

2D Physics

- Position:
  (x,y)
- Orientation:
  (α) – angle (scalar)

3D Physics

- Position:
  (x,y,z)
- Orientation:
  quaternion or
  axis,angle or
  axis * angle or
  3x3 matrix or
  Euler angles

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Newtonian dynamics: summary

Current object location
Position p p = (x,y,z)

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Newtonian dynamics: summary

Current object location	Rate of change of ← (d / dt)	← "with mass" (momentum)	What changes the rate of change (d² / dt²)	← "with mass"
Position p p = (x,y,z)	Velocity $\vec{v}$ $\vec{v} = \dot{p}$ ($\lvert\vec{v}\rvert$ = "speed")	Momentum $\vec{v} \cdot m$	Acceleration $\vec{a} = \dot{\vec{v}} = \ddot{p}$	Force $\vec{f}$ $\vec{f} = \vec{a} \cdot m$
Orientation (e.g. quaternion)	Angular velocity $\vec{\omega}$	Angular momentum $\vec{\omega} \cdot I$ I = moment of inertia (for axis) ("rotational inertia")	Angular acc. $\vec{\alpha}$	Torque $\vec{\tau}$ $\vec{\tau} = \vec{\alpha} \cdot I$ ("mechanic momentum")

state (is kept! inertia!)
(changes, but only continuously)

change the state
(no memory)

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Per-object constant: mass & its distribution (for non point-shaped ones)

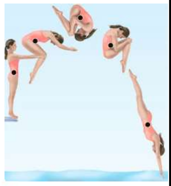
A few quantities associated to each object

- constants: they don't (usually) change
- they are *input* of the physics dynamic simulation

- Mass:
  - resistance to change of velocity

Distribution of mass:

- Moment of Inertia:
  - resistance to change of *angular* velocity



- Barycenter:
  - the center of mass

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Mass: notes

- resistance to change of velocity
  - *inertial* mass
- also, incidentally:
  ability to attract every other object
  - *gravitational* mass
  - happens to be the same
- it's what you measure with a scale
- Unity of measure:
  kg, g…

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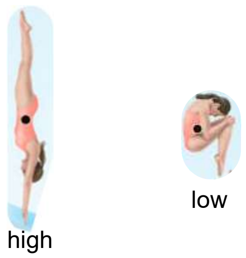
Moment of inertia: notes 1/2

- Resistance to change of angular velocity



- (an object rotates around its barycenter)

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Moment of inertia: notes 2/2

- Scalar moment of inertia
  - Resistance to change of angular velocity
  - Depends on the mass, and on its *distribution*
    - the farthest one sub-mass from the axis, the > the resistance
  - In 3D: it's different for each axis of rotation
    - It can be computed for any axis, thanks to…
- In 3D: moment of inertia as a 3x3 Matrix
  - a matrix **A** used to extract that scalar, for any given axis
  - given an axis **a** (**a** = unit vector), the *moment of inertia* is

$$a^T A a$$

  - matrix **A** can be computed, once and for all, for a rigid object
    - how: that's beyond this course
    - in practice: use given formulas for common shapes
    - or sum the contributions for each sub-mass

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Barycenter: notes

- Aka the center of mass
  - a position
- In the discrete setting:
  simply the *weighted average* of the positions of the subparts composing an object
  - literally "weighted": with their masses
- Does not necessarily coincide with the origin of the local frame of that object
  - but it can

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State of a rigid object in a physical simulation

```
Point position
Rotation orientation
```
current

```
Vector velocity
Vector angular_velocity
```
current rates of change

} updated by physics (dynamics)

```
Scalar mass
Matrix moment_of_inertia
Point barycenter
Scalar drag
…
```
constants

drag ← frictions; see later

setup at initialization, (rarely) changed e.g. by scripts

Note: acceleration/forces/torques are ***not*** *part of the state*

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In unity

```
Point position
Rotation orientation
```

part of Transform component

```
Vector velocity
Vector angular_velocity
Scalar mass
Matrix moment_of_inertia
Point barycenter
Scalar drag
bool isKinematic
```

the RigidBody component

Adding a "RigidBody" component to a Game Object is to say: *"please let the Phys. engine take care of this object"*

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In unity (using Unity terminology)

note: they are the components of the **global** transformation!

```
Vector3 position
Quaternion rotation
```

part of Transform component

note: speed = velocity.magnitude

```
Vector3 velocity
Vector3 angular_velocity
float mass
Vector3 inertiaTensor
Quaternion inertiaTensorRotation
Vector3 centerOfMass
float drag
bool isKinematic
```

the RigidBody component

per second (not per frame!)

moment of inertia matrix

the Vector3 = a diagonal matrix D by rotating it $R^T D R$ → the final matrix

the barycenter (in local space)

if true: disable dynamics (but keeps e.g. collisions)

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State of a particle (point sized obj) in a physical simulation

`Point` **`position`**

~~`Rotation` **`orientation`**~~

`Vector` **`velocity`**

~~`Vector` **`angular_velocity`**~~

`Scalar` **`mass`**

~~`Matrix` **`moment_of_inertia`**~~

~~`Point` **`barycenter`**~~

`Scalar` **`drag`**

…

not used for point sized objects!

One possibility in a game phys engine is to only simulate point-particles.

Simpler: no rotation needed!

We will see later how to still get rigid bodies back.

For now, we focus on this simpler case.

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Newtonian Dynamics (for particles)

describes the forces given all the particle positions (and more)

$$\begin{cases} \vec{f}(t) = \text{function}(\, \mathbf{p}(t)\, , \dots) \\ \vec{a}(t) = \dfrac{\vec{f}(t)}{m} \\ \vec{v}(t) = \vec{v}_0 + \displaystyle\int_{t'=0}^{t} \vec{a}(t') \cdot dt' \\ \mathbf{p}(t) = \mathbf{p}_0 + \displaystyle\int_{t'=0}^{t} \vec{v}(t') \cdot dt' \end{cases}$$

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Newtonian Dynamics: equivalent formulation

$$
\begin{cases}
\vec{f}(t) = \text{function}(\,\mathbf{p}(t)\,,\dots) \\
\vec{v}(t) = \dot{\mathbf{p}}(t) \\
\vec{a}(t) = \ddot{\mathbf{p}}(t) = \dfrac{\vec{f}(t)}{m} \\
\dot{\mathbf{p}}(0) = \vec{\mathrm{v}}_0 \\
\mathbf{p}(0) = \mathrm{p}_0
\end{cases}
$$

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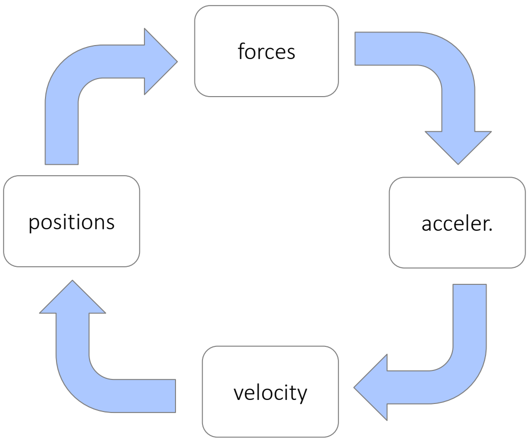
Dynamics (Newtonian)



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An obvious remark, but

Simulation time ≠ Wall time

the t in all the slides

They are just artificially made to flow in sync… usually

- But (e.g.) not when:
  game is paused (t is constant), replays, fast forwards, reverses…

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An obvious remark, but

Simulation time ≠ Wall time

the t in all the slides

Occasionally, the difference is spectacularly exploited by clever gameplay designs!

PoP – the sands of times serie (Ubisoft, 2003-now)

Braid (Jonathan Blow, 2008)

The longing (Studio Seufz, 2020)

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Computing physics evolution

- Analytical solutions:

  state = function(t)

  Given force functions (and acc), find the functions (pos, vel,...) in the specified relations:

$$\begin{cases} \vec{f}(t_C) = funz(p(t_C),\ldots) \\ \vec{a}(t_C) = \vec{f}(t_C)/m \\ \vec{v}(t_C) = \vec{v}_0 + \int_0^{t_C} \vec{a}(t)\cdot dt \\ p(t_C) = p_0 + \int_0^{t_C} \vec{v}(t)\cdot dt \end{cases}$$

- Numerical solutions:

  1. state$_{(t=0)}$ ← init
  2. state$_{(t+1)}$ ← do_1_step(state$_t$)
  3. goto 2

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Analytical solutions

$$\mathbf{p}(t) = \text{some function of } t$$

derivative w.r.t. time

$$\vec{v}(t) = \dot{\mathbf{p}}(t)$$

$$\vec{a}(t) = \ddot{\mathbf{p}}(t) = forces(\,\mathbf{p}(t)\,, \dot{\mathbf{p}}(t), t, \ldots)/m$$

$$\dot{\mathbf{p}}(0) = \vec{\mathrm{v}}_0$$
$$\mathbf{p}(0) = \mathrm{p}_0$$

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Analytical solutions

Find the positions as a functions $\mathbf{p}(t)$ of time t such that...

a given function

$$\ddot{\mathrm{p}}(t) = \mathrm{forces}(\,\mathrm{p}(t), \dots)/\mathrm{m}$$

$$\dot{\mathrm{p}}(0) = \vec{\mathrm{v}}_0$$
$$\mathrm{p}(0) = \mathrm{p}_0$$

sometimes, a function of other things too (e.g. velocity, time...). Harder to solve!

the initial conditions (we want to find their evolution!)

A system of ODE
(Ordinary Differential Equation)

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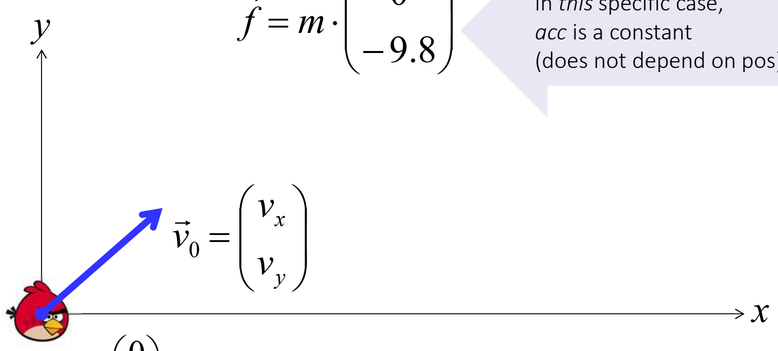
Simple example: analytical solution

«ballistic shooting» of a mass, in 2D, ignoring friction...

$$\vec{f} = m \cdot \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$$

in *this* specific case, *acc* is a constant (does not depend on pos)



$$\vec{v}_0 = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$p_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

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Simple example: analytical solution

Solving...

$$\vec{f}(t_C) = fun(p(t_C),\dots)$$
$$\vec{a}(t_C) = \vec{f}(t_C)/m$$
$$\vec{v}(t_C) = \vec{v}_0 + \int_0^{t_C} \vec{a}(t)\cdot dt$$
$$p(t_C) = p_0 + \int_0^{t_C} \vec{v}(t)\cdot dt$$

$$\vec{f}(t_C) = m \cdot \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$$

$$\vec{a}(t_C) = \vec{f}(t_C)/m = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$$

$$\vec{v}(t_C) = \begin{pmatrix} v_x \\ v_y \end{pmatrix} + \int_0^{t_C} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \cdot dt = \begin{pmatrix} v_x \\ v_y - 9.8 \cdot t_C \end{pmatrix}$$

$$p(t_C) = p_0 + \int_0^{t_C} \vec{v}(t)\cdot dt = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \int_0^{t_C} \begin{pmatrix} v_x \\ v_y - 9.8 \cdot t \end{pmatrix} \cdot dt = \begin{pmatrix} v_x \cdot t_C \\ v_y \cdot t_C - 9.8/2 \cdot t_C^{\,2} \end{pmatrix}$$

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Simple example: analytical solution

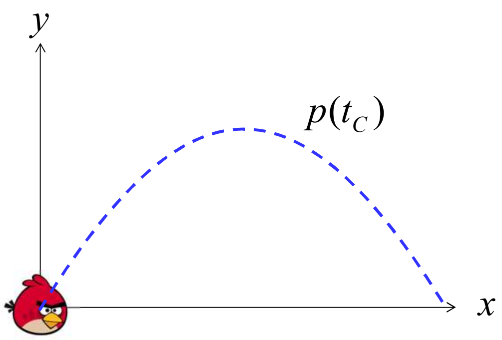
Final result:

$$\vec{f}(t_C) = m \cdot \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$$

$$\vec{a}(t_C) = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$$

$$\vec{v}(t_C) = \begin{pmatrix} v_x \\ v_y - 9.8 \cdot t_C \end{pmatrix}$$

$$p(t_C) = \begin{pmatrix} v_x \cdot t_C \\ v_y \cdot t_C - 9.8/2 \cdot t_C^{\,2} \end{pmatrix}$$



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Numerical integration

$$\vec{f}(t_C) = \text{function}(p(t_C), \dots)$$
$$\vec{a}(t_C) = \vec{f}(t_C)/m$$
$$\vec{v}(t_C) = \vec{v}_0 + \int_0^{t_C} \vec{a}(t) \cdot dt$$
$$p(t_C) = p_0 + \int_0^{t_C} \vec{v}(t) \cdot dt$$

It's our way to solve the ODE

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Numerical integration

- A numerical integrator computes the integral as summed area of small rectangles
  - For a physics engine, this means just updating velocity and positions at each physics step
- A crucial parameter is the width of the rectangles i.e. *dt* = the duration of the physics step (in virtual time)
  - If physics system perform *N* steps per second: *dt* = 1.0 sec / N
  - *N* is not necessarily same rendering frame rate e.g.: rendering 30 FPS but physics: 60 steps per seconds
  - *dt* is not necessarily constant during the simulation (but in most system, it is)

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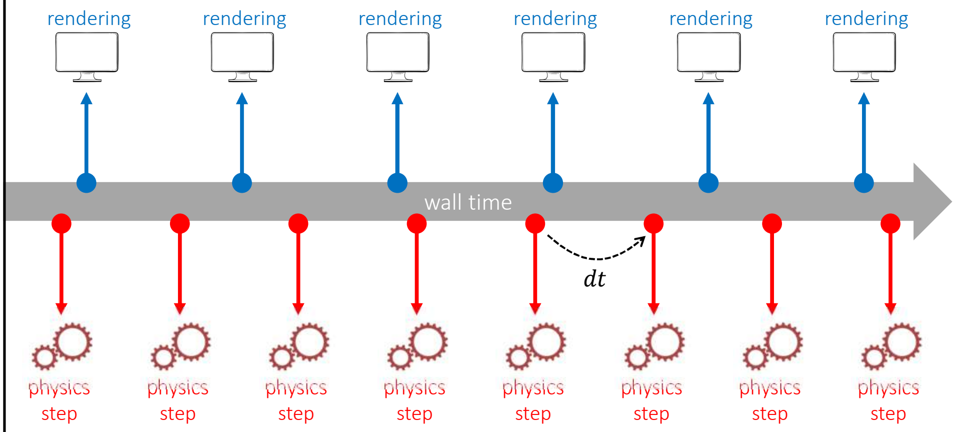
Rendering *Frames-per-Seconds* (FPS) vs Physics *Steps-per-Seconds*



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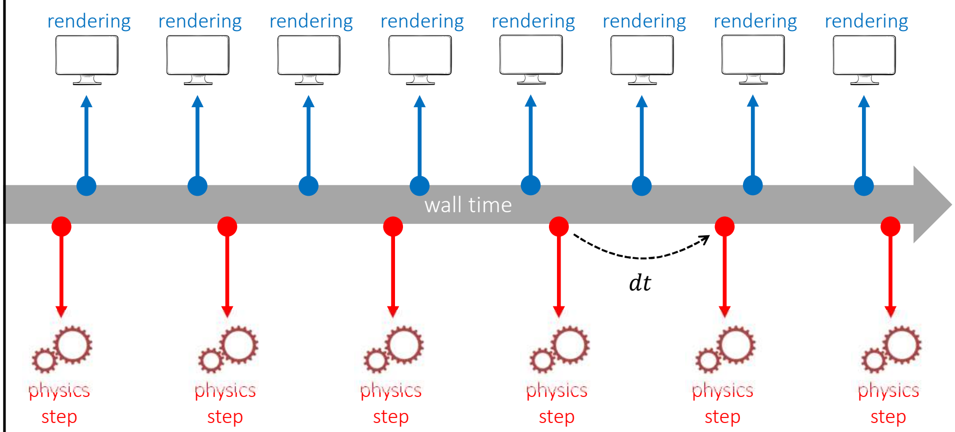
Rendering *Frames-per-Seconds* (FPS) vs Physics *Steps-per-Seconds*



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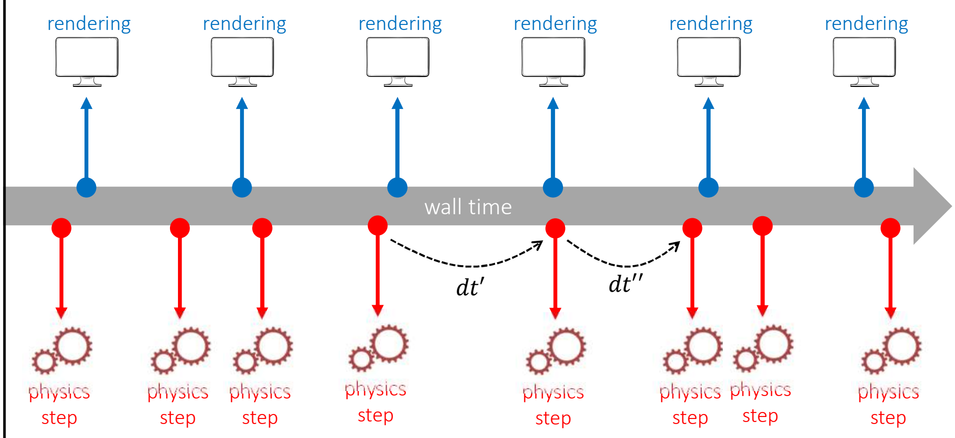
Variable timesteps?



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Numerical methods: features

- How efficient / expensive
  - must be at least soft real-time
    - (if from time to time computation delayed to next frame, ok)
- How accurate
  - must be at least plausible
    - (if stays plausible, differences from reality are acceptable)
- How robust
  - rare completely wrong results
    - (and never crash)
- How generic
  - Which phenomena / constraints / object types is it able to recreate?
  - requirements depend on the context (ex: gameplay)

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Euler integration methods

For each step:

$$\vec{f} = fun(\mathrm{p}, \dots)$$

(1) Evaluate the force on each particle as a function of positions (of this and other particles) and maybe other things too

$$\vec{a} = \vec{f}/m$$

(2) acceleration of each particle given by: total force on it divided by its mass

$$\vec{v} = \vec{v}_0 + \int \vec{a} \cdot dt$$

(3) Update velocity with acceleration

$$p = p_0 + \int \vec{v} \cdot dt$$

(4) Update position with velocity

(state) , (temp variables)

Assumption: a

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Euler integration methods

init state

$$\mathbf{p} \leftarrow \cdots$$
$$\vec{\mathrm{v}} \leftarrow \cdots$$

one step

$$\vec{\mathrm{f}} \leftarrow fun(\mathbf{p}, \dots)$$
$$\vec{\mathrm{a}} \leftarrow \vec{\mathrm{f}}/m$$
$$\mathbf{p} \leftarrow \mathbf{p} + \vec{\mathrm{v}} \cdot dt$$
$$\vec{\mathrm{v}} \leftarrow \vec{\mathrm{v}} + \vec{\mathrm{a}} \cdot dt$$

$$t = t + dt$$

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Forward Euler *pseudo code*

```
Vec3 position = …
Vec3 velocity = …

void initState(){
   position = …
   velocity = …
}

void physicStep( float dt )
{
   Vec3 acceleration = compute_force( position ) / mass;
   position += velocity     * dt;
   velocity += acceleration * dt;
}

void main(){
  initState();
  while (1) do physicStep( 1.0 / FPS );
}
```

Equivalent to…

$$\vec{f}_i = function(p_i, \dots)$$
$$\vec{a}_i = \vec{f}/m$$
$$\vec{v}_{i+1} = \vec{v}_i + \vec{a}_i \cdot dt$$
$$p_{i+1} = p_i + \vec{v}_i \cdot dt$$

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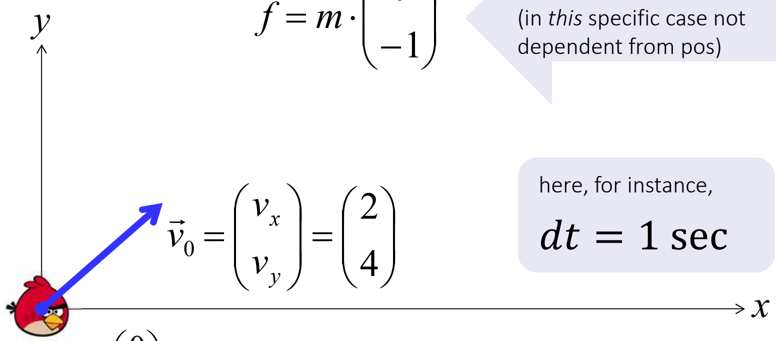
Simple example: numerical solution

Same phenomena of previous example

$$\vec{f} = m \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

constant (in *this* specific case not dependent from pos)



$$\vec{v}_0 = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$p_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

here, for instance, $dt = 1 \text{ sec}$

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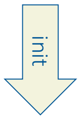
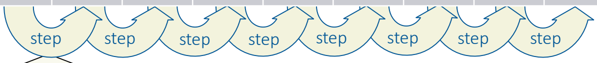
Simple example: numerical solution



Time:	0	1	2	3	4	5	6	7	...
vel:	(2,3)	(2,2)	(2,1)	(2,0)	(2,-1)	(2,-2)	(2,-3)	(2,-4)	...
pos:	(0,0)	(2,3)	(4,5)	(6,6)	(8,6)	(10,5)	(12,3)	(14,0)	...



$\vec{f} = m \cdot \binom{0}{-1}$

$\vec{a} = \vec{f}/m$

$\vec{v} = \vec{v} + \vec{a} \cdot dt$

$p = p + \vec{v} \cdot dt$

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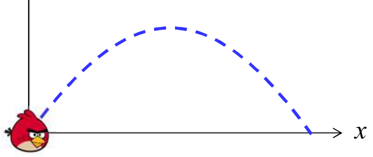
Physics evolution computation

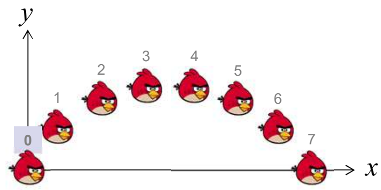
- Analytical solutions:

$\binom{p_x}{p_y} = function\_pos(time)$

$\binom{v_x}{v_y} = function\_vel(time)$



- Numerical solutions:



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Physics evolution computation

- Analytical solutions:
  - Super efficient!
    - Close form solution
  - Accurate
  - Only simple systems
  - formulas found case by case (often not existing!)
  - NO (but, for instance, useful to allow the AI to make predictions)
- Numerical solutions:
  - Expensive (iterative)
    - but *interactive*
  - Integration errors
  - Flexible
  - Generic
  - YES

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Integration errors

- A numerical integrator only approximates the real value of the integrals
- The discrepancy (simulation errors) accumulate with virtual time during all the simulation
- How much error is accumulated?
- It depends on *dt* !
  - Small *dt* ⇒ more steps needed (for same virtual time) ⇒ more computationally expensive, but smaller errors, i.e. more accurate simulation

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Order of convergence

- How much does the total error decrease as *dt* decreases?
  - That's called the Order of the simulation
  - 1st order: the total error can be as large as O(dt^1)
    - "if the number of physics steps doubles (physical computation effort doubles) *dt* becomes halves and errors can be expected to halve"
    - The error introduced by each single step is O(dt^2),
  - The Euler seen is 1st order
    - This is not too good, we want better
    - Note: The error is usually not that bad as linear with *dt*, but they *can* be

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The integration steps *dt* of any numerical methods (summary)

dt : delta of virtual time from last step
  - the "temporal resolution" of the simulation!
- if large: more efficiency
  - fewer steps to simulate same amount of virtual time
- if small: more accuracy
  - especially with strong forces and/or high velocities
- Common values: 1 sec / 60 … 1 sec / 30
  - i.e. a step simulates around 16 … 32 msec. of virtual time
  - note: it's not necessarily the same refresh rate of rendering (FPS of rendering ≠ FPS of physics. Rendering can be *less*!)
  - note: di dt is not necessarily the same in all physics steps (need more accuracy *now*? Decrease *dt*

number of physics steps per sec, or «physics FPS»

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