Course Plan

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- lec. 2: Mathematics for 3D Games
- lec. 3: Scene Graph
- lec. 4: Game 3D Physics +
- lec. 5: Game Particle Systems
- lec. 6: Game 3D Models
- lec. 7: Game Textures
- lec. 9: Game Materials
- lec. 8: Game 3D Animations
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Changing the value of dt in Verlet

(whenever it's not constant)

Problem:

if dt now changes to a new dt'

then, all $\mathbf{p}_{old}$ must be updated to some $\mathbf{p}'_{old}$

Find $\mathbf{p}'_{old}$:

$$\vec{v} = (\mathbf{p}_{now} - \mathbf{p}_{old})/dt$$
$$\vec{v} = (\mathbf{p}_{now} - \mathbf{p}'_{old})/dt'$$

current velocity $\vec{v}$ and position $\mathbf{p}_{now}$ must not change

$$\Longrightarrow$$

$$\mathbf{p}'_{old} = \mathbf{p}_{now} \cdot (dt - dt')/dt + \mathbf{p}_{old} \cdot dt'/dt$$

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Velocity damping in Verlet

- Velocity at next frame: $\vec{v} = (\mathbf{p}_{next} - \mathbf{p}_{now})/dt$ (implicit)
- We want to multiply $\vec{v}$ a factor c_{DAMP} (e.g. 0.98 obtained as $(1\text{-}dt \cdot c_{\text{DRAG}})$)
  - before applying accelerations
- We can do that using a more general formula for $\mathbf{p}_{next}$

$$\mathbf{p}_{next} = 2 \cdot \mathbf{p}_{now} - 1 \cdot \mathbf{p}_{old} + dt^2 \cdot \vec{a}$$

$$\mathbf{p}_{next} = (1 + c_{\text{DAMP}}) \cdot \mathbf{p}_{now} - c_{\text{damp}} \cdot \mathbf{p}_{old} + dt^2 \cdot \vec{a}$$

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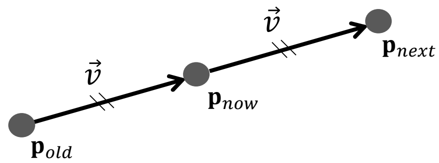
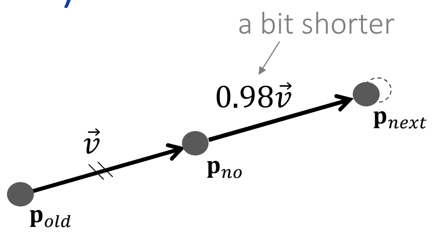
Velocity damping in Verlet (geometric interpretation)





$$\mathbf{p}_{next} = 2 \cdot \mathbf{p}_{now} - 1 \cdot \mathbf{p}_{old}$$

Equivalently,
$\mathbf{p}_{next}$ is an extrapolation of $\mathbf{p}_{now}$, $\mathbf{p}_{old}$:

$$\mathbf{p}_{next} = mix(\mathbf{p}_{old}, \mathbf{p}_{now}, 2)$$

$$\mathbf{p}_{next} = 1.98 \cdot \mathbf{p}_{now} - 0.98 \cdot \mathbf{p}_{old}$$

Equivalently,
$\mathbf{p}_{next}$ is a different extrapolation of $\mathbf{p}_{now}$, $\mathbf{p}_{old}$:

$$\mathbf{p}_{next} = mix(\mathbf{p}_{old}, \mathbf{p}_{now}, 1.98)$$

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Verlet integration + "Position Based Dynamics" (PBD)

init state

$\mathbf{p}_{now} \leftarrow \dots$

$\mathbf{p}_{old} \leftarrow \dots$

one step

$\vec{f} \leftarrow fun(\mathbf{p}_{now})$

$\vec{a} \leftarrow \vec{f}/m$

$\mathbf{p}_{next} \leftarrow 2\mathbf{p}_{now} - \mathbf{p}_{old} + \vec{a} \cdot dt^2$

Enforce constraints on $(\mathbf{p}_{next})$

$\mathbf{p}_{old} = \mathbf{p}_{now}$

$\mathbf{p}_{now} = \mathbf{p}_{next}$

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Position Based Dynamics (PDB)

- A positional constraint is an equality/inequality involving the *positions* of particles.
  (a formula with '=' '>' '<' etc.)
  - Useful, for example, to model consistency conditions
  - Like "*solid objects don't compenetrate each other*", or "*steel bars won't become shorter or longer than they are*"
  - We will see many examples
- We enforce (impose) positional constraint directly by displacing the *positions* of particles
  - Thanks to Verlet: this displacement automatically causes some appropriate update of the velocity!
  - it's not necessarily correct, but it's plausible and robust

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Verlet + Position Based Dynamics. Advantages

- flexibility: different constraints can be used to model many different phenomena
  - Useful constraints are straightforward to define
  - They are easy to impose (they involve only few particles)
  - They can be used to model many possible phenomena
  - See following slides for examples
- robustness : plausibility is ensured by *explicitly* enforcing the conditions we want to see
  - For example: a ball won't ever be seen outside the box containing it (at lest, not for many frames)
- No forces / impulses are needed to enforce any such consistency conditions
  - Which is what actually happens in the real world, but is more difficult to simulate robustly

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Example of a positional constraint

«I want all particles to stay above ground (their y is not negative) »

Enforce this constraint: trivial!

```
for (each particle i)
{
  if (p[i].y < 0) p[i].y = 0;
}
```

Imposing constraints like this one is a first part of collision response. For re-bounces, impulses must still be added (see collisions).

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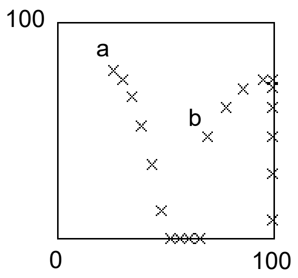
Example of a positional constraint (here, in 2D physics)

«I want particles to stay inside a 2D box [0 – 100] x [0 – 100] *»*



Enforce this constraint: simple clamp!

```
for (each particle i)
{
  p[i].x = clamp( p[i].x, 0, 100 );
  p[i].y = clamp( p[i].y, 0, 100 );
}
```

⚠ Imposing constraints like this one is a first part of collision response. For re-bounces, impulses must still be added (see collisions).

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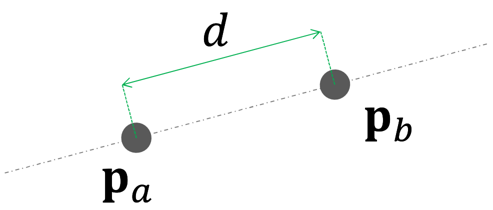
Example of positional constraint: equidistance constraint

*«Particles **a** and **b** must stay at a fixed distance **d** »*



I want that... $\|\mathbf{p}_a - \mathbf{p}_b\| = d$

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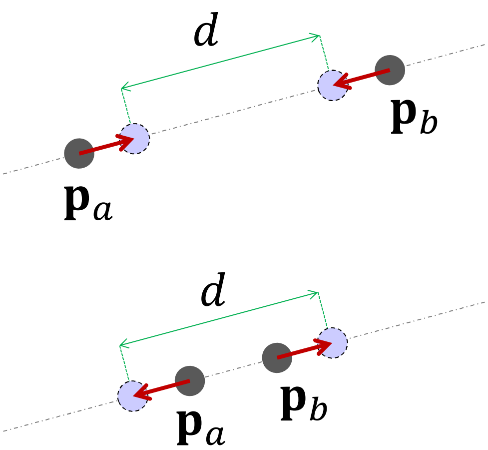
Enforce equidistance constraints (assuming equal masses)

if $\|\mathbf{p}_a - \mathbf{p}_b\| > d$

if $\|\mathbf{p}_a - \mathbf{p}_b\| < d$



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Enforce equidistance constraints: pseudo code

```
Vector3 pa, pb; // curr positions of a,b
float d;        // distance (to enforce)

Vector3 v = pa - pb;
float currDist = v.length;

v /= currDist;  // normalization of v

float delta = currDist - d ;

pa += ( 0.5 * delta) * v;
pb -= ( 0.5 * delta) * v;
```

assuming equal mass, we move each particle *half the way*
(see later for the more general case)

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Compare: equidistance constraints vs. springs

some constant scalar parameter l

- Similar
  - they both mean:
    these 2 particles "want to be" at *this* distance (not more, not less)
- but different
  - equidistance constraint:
    - applied during constraint enforcement
    - directly affects positions
    - models a rigid rod between the two particles
      - of a given length
    - sometimes called a "HARD" constraint
  - spring:
    - applied during force evaluation step
    - affects forces, therefore accelerations
    - models a deformable spring between the two particles
      - of a given length
    - sometimes called a "SOFT" constraint
- A physic engine can combine them in one object!

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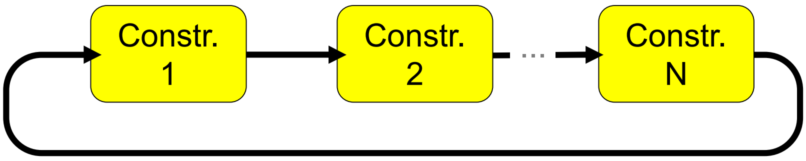
Enforcing sets of constraints

- There are many constraints to impose:
  when you solve one → maybe you break another!
- Simultaneous enforcement: computationally expensive
- Practical & easy solution: just enforce them in cascade
  (similar in concept to Gauss-Seidel solvers):



Repeat until convergence (= max error below threshold)
...but at most for *N* times! (reminder: our simulation is *soft* real-time)

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Enforcing sets of constraints one after the other (in cascade)

- The whole loop for imposing the constraints happen in the constraint enforcement phase on one physics step!
- Notes about convergence:
  - needed iterations (typically) few: e.g. 1 ~ 10 (efficient!).
  - if convergence not reached within a given number of steps: never mind, next frames will fix it (it’s fairly robust)
  - (it is never reached, if constraints are contradictory)
  - Optimization (to reduce the number of needed iterations): solve the most unsatisfied constraints first

⚠ Problem: it’s a sequential approach! ☹
  - parallelized versions (similar to Jacobi solvers) are possible
  - they have a worse convergence in practice (they require more iterations), but each iteration is faster

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Compounds of particles disguised as rigid bodies

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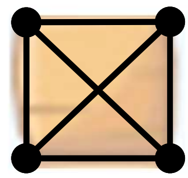
Combining equidistance constraints we obtain rigid objects

- Rigid body dynamics as emerging behavior
  - without explicitly keeping track their orientation, angular vel, angular acc., etc.



A box?
(rigid object)
In 2D a configuration of:
- 4 particles
- 6 equidistance constraints

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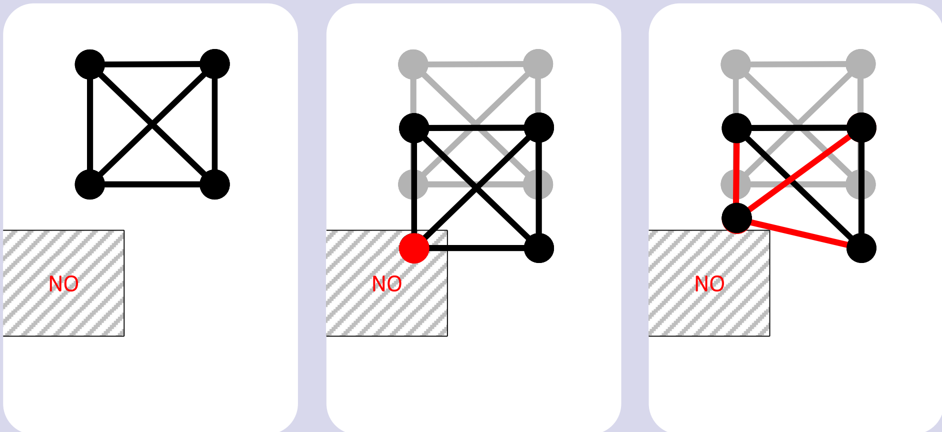
Example



FRAME 0

FRAME 1
before constraints

FRAME 1
after 1st constraint

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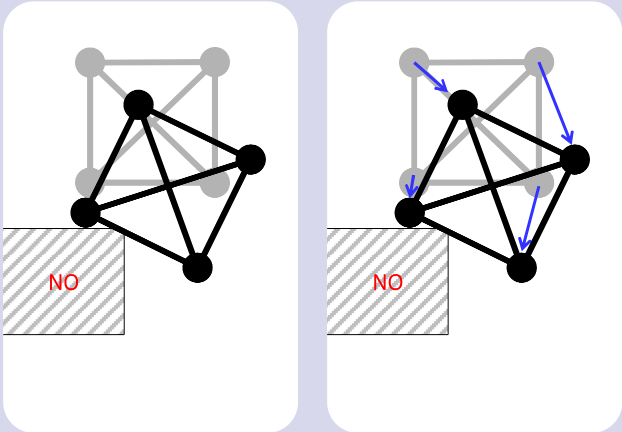
Example



FRAME 1
after all constraints
multiple times

FRAME 1
resulting
(implicit) velocities

In total: the "box",
under gravity + collision
- had rotated
- gained angular velocity (will keep rotating by inertia)

even the system does not (explicitly) handle rotations or angular velocities

(works in 3D as well!)

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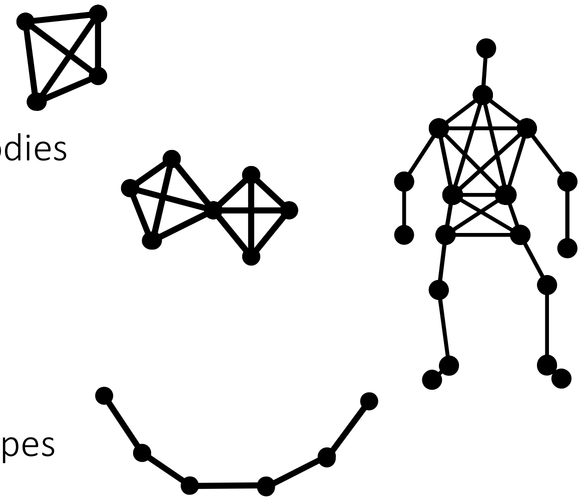
We can combine equidistance constraints to obtain…

- Rigid bodies
- Articulated bodies
- Ragdolls
- Cloth
- Non-elastic ropes
- …and more



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Enforcing a positional constraint:
(assuming for now all particles have same mass)

- Test: does the equality/inequality hold?
- If so: nothing to do!
- Else:
  - All particles must be displaced a bit, so that it will
  - Infinite ways to achieve this. Which one to pick?
  - Answer:
    minimize the sum of *squared* displacements
    (with respect to current position)
  - Find the minimizer by analytically
    solving simple math problems
    ("analytically" = in closed form = "with formulas")

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Enforcing positional constraints
(assuming for now all particles have same mass)

- We want to enforce a constraint $\mathcal{C}$ on particles $\mathrm{a}, \mathrm{b}, \mathrm{c}, \ldots$ in positions $\mathbf{p}_\mathrm{a}, \mathbf{p}_\mathrm{b}, \mathbf{p}_\mathrm{c}$ …
- $\mathcal{C}$ defined as an equality/inequality of $\mathbf{p}_\mathrm{a}, \mathbf{p}_\mathrm{b}, \mathbf{p}_\mathrm{c}, \ldots$:

$$\mathcal{C}: (\mathbf{p}_\mathrm{a}, \mathbf{p}_\mathrm{b}, \mathbf{p}_\mathrm{c}, \ldots) \rightarrow \{\, true \,, false \,\}$$

- We must apply the displacements $\vec{d}_\mathrm{a}$, $\vec{d}_\mathrm{b}$, $\vec{d}_\mathrm{c}$ found by:

$$\underset{\vec{d}_\mathrm{a}, \vec{d}_\mathrm{b}, \vec{d}_\mathrm{c}, \ldots}{\operatorname{argmin}} \left(\|\vec{d}_\mathrm{a}\|^2 + \|\vec{d}_\mathrm{b}\|^2 + \|\vec{d}_\mathrm{c}\|^2 + \cdots \right)$$

$$\text{such that } \mathcal{C}(\mathbf{p}_\mathrm{a} + \vec{d}_\mathrm{a}, \mathbf{p}_\mathrm{b} + \vec{d}_\mathrm{b}, \mathbf{p}_\mathrm{c} + \vec{d}_\mathrm{c}, \ldots)$$

among all the choices that satisfy this,
we want the one which minimizes this

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Positional constraint example: "please don't sink under a general plane"

- We want to enforce the constraint
  "particle a must be above a given constant plane"
  - Given: position of the particle $\mathbf{p}_a$ and its mass m_a
  - A plane given by a point on it $\mathbf{p}_q$ and its normal $\hat{n}_q$
- We need to apply the displacement $\overrightarrow{d_a}$ found by minimizing:

$$\underset{\overrightarrow{d_a}, \overrightarrow{d_b}}{\operatorname{argmin}} \left(\left\| \overrightarrow{d_a} \right\|^2 \right)$$

$$\text{such that } \left\| (\mathbf{p}_a - \mathbf{p}_q) \cdot \hat{n}_q \right\| > 0$$

- And the solution (in closed form) is, trivially…

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In pseudocode

```
Vector3 pa; // curr positions of a
float ma;   // mass (no effect here)
Vector3 pq; // point on the plane
Vector3 nq; // normal of the plane (unit)

Vector3 v = pa – pq;
float currDist = Vector3.dot( v , n );

if (currDist < 0.0)
   pa -= currDist * n; // just project!
else {} // constrain ok, nothing to do
```

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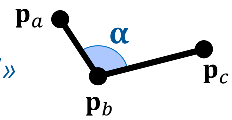
More examples of possible positional constraints

- Preserve volume of some object: *«Volume is* v_{CONST} *»*
  - How to impose it:
    1. Estimate current total volume v
    2. uniform scale the entire object by factor $\sqrt[3]{v_{\text{CONST}} / v}$
- Fixed positions: *«particle* a *stays in* $\mathbf{p}_\mathbf{a}$ *»*
  - the particle is "pinned" in position
  - trivial to impose, but still useful!
- Angle constraints, e.g. $\boldsymbol{\alpha} < \boldsymbol{\alpha}_{\text{max}}$
  - e.g., on joints: *«elbows cannot bend backward»*



- Coplanarity / collinearity
  - «these N particles must stay on a line / on a plane»
- Non interpenetration
  - part of collision handling – see collisions later

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Position Based Dynamics (PBD) summary

- A general approach for computing dynamics
- Ingredients:
  1. Use Verlet integration on particles
     - velocities are implicit
     - changes in positions induce changes in velocities
  2. Implement positional constraints on particles
     (e.g., equidistance constraint) to model things like:
     - Rigid bodies
     - Articulated / non rigid bodies
     - Non penetration (maybe, add collision impulses, see later)

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