3D video games

Collision Handling

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Course Plan

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- lec. 2: Mathematics for 3D Games
- lec. 3: Scene Graph
- lec. 4: Game 3D Physics +
- lec. 5: Game Particle Systems
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Collision Handling: a preliminary consideration

- Two types of objects in a game:
  - static
    - Never moves (speed = 0)
    - Part of the setting, background
    - Affects other objects, not affected by other objects
  - non-static
    - Can move around (for any reason)
- Two types of collisions:
  - one-way : a non-static object with a static object
  - two-ways : a non-static object with a non-static object

	Static	Movable
Static	🚫	One Way
Movable	One Way	Two Ways

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Collision Handling: a preliminary consideration

One way:
- easier detection
- easier response

Two ways:
- costly detection
- complex response

By labelling every object as static / movable, I reduce the needed computation considerably!
E.g., if 50% static, 50% movable then
- ¼ of the potential collisions cease to exists. Of the rest:
- 2/3 are one ways
- Only 1/3 are two-ways

Static VS static: no collision handling.
Not just an “optimization”, but a feature:
- Wall models can compenetrate, to build a house (no collision)
- Buildings can sink into the terrain (no collision)
- Etc.

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Collision Handling

- Collision detection
  - find out when they occur
- Collision response
  - compute their effects

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Collision response

- Enforce non-penetration
  - objects must be placed in valid positions
  - (*when to:* always)
- Impacts
  - with impulses (bounces)
  - (*when to:* collision occurred now, but not in the pref frame)
- Frictions between the two objects
  - energy dissipation
  - (*when to*: from 2° consecutive step of collision)
- Ad-hoc effects
  - breaking objects, gameplay effects (HP loss?), etc (by scripts)
  - (*when to - if at all:* entirely gameplay dependent)

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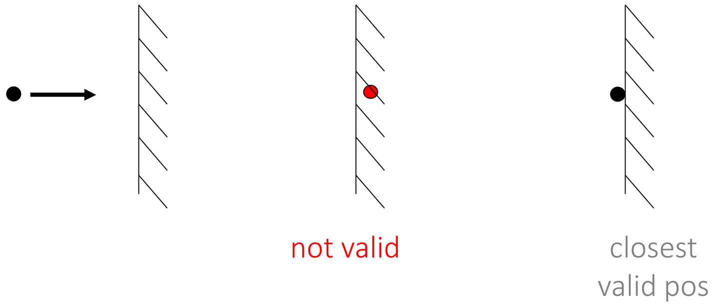
Enforcing non-penetration

- Invalid position?
  - strategy 1: revert to last valid pos (easy to do, not ideal)
  - strategy 2: project to closest valid pos (necessary, in PBD)



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Enforcing non-penetration

- In PBD:
  just another positional constraint
  - bonus: velocity updates
    (similar to inelastic impacts)
  - but we will need to explicitly compute impacts if we want a better control of the behavior

Note: asymmetrical constraint (> not =)

A big practical problem ☹ : the presence of the constraint it is not known a-priori.

- How to enforce this constraint:
  - *two-ways* :
    displace both of them,
    minimizing the summed squared displacements × the mass
  - *one-way* :
    only displace the one movable objects by the minimal amount
    (equivalent to the above, when fixed object mass → ∞)

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Friction

- Apply on prolonged contact
  - collision with an object that was colliding last frame too
- Affects component of velocity parallel to contact plane
- Can be implemented with: (1) forces, or (2) velocity damping
- Forces:
  - Opposite to current velocity, *projected on contact plane* (note: I need its normal)
  - Magnitude: proportional to speed

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Resolving the impacts

so, it's the effect of an impulse

- Sudden velocity change
  - resolve the impact = determine the new velocities $\vec{v}_{new}$
  - equivalently, determine the impulses $\vec{\imath} = (\vec{v}_{new} - \vec{v}_{old}) \cdot m$
- *All* impacts preserve total momentum $m \cdot \vec{v}$
  - *Always*, no matter what

a vector (ita: *«quantità di moto»*)

- To resolve the impact, we need further assumptions, different for each type of the impact:
  - elastic
  - inelastic
  - or anything in between

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Different type of impacts

(completely)
elastic
impact

(completely)
inelastic
impact

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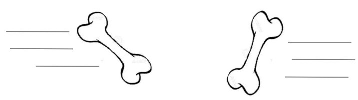
“Bounciness” (or impact elasticity)

“Bounciness” = 1.0

…



“Bounciness” = 0.5

…

“Bounciness” = 0.0

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"Bounciness" (or impact elasticity)

- Elastic impact: no energy lost
- Inelastic impact: energy losses
  - e.g. objects are damaged, heat is produced...
- "Bounciness":
  a (made up) property of physical objects in games
  - It models the behavior of the object under impacts,
    as a mix between the two extreme behaviors above
  - Associated by designers to all virtual objects in the game
- Note: nothing of this is how stuff really works!
  - not even for the two extremes
  - it's an approximation (especially for mixed bounciness)
  - Remember: we are just going for plausibility

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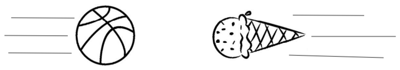
What about this impact?



"Bounciness" = ???

- Practical solution:
  adopt some formula between the
  bounciness values associated to the two objects
  - For example: avg, min, max
  - It's a choice of the game engine
  - (can be hard-wired in the physics engine,
    or exposed to the users)

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The assumptions for the two types of impact

- (completely) elastic impact
  - preservation of total kinetic energy $\frac{1}{2} m \cdot \|\vec{v}\|^2$ (a scalar)
  - impulse direction = the normal of impact point
- (completely) inelastic impact
  - after the impact, the two bodies have the same velocity
  - (as if the impact momentarily glued them together)
    (they will still move apart in subsequent frames)
- mixed cases:
  - solve for both cases, interpolate resulting velocities
  - interpolation weight is the "bounciness"

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The assumptions for the two types of impact

	Assumptions		
Elastic	After the impact, the total energy is the same as before	The impulse is in the direction of the impact normal	...and the total momentum is the same as before
Inelastic	After the impact, the two bodies share the same velocity		...and the total momentum is the same as before

Remember the impulse (force x time) is the (instantaneous) change of momentum.
This is a way to say that the total impulse is zero
$i_A + i_B = 0$
aka
$i_A = -i_B$
aka the 3rd law of dynamics.

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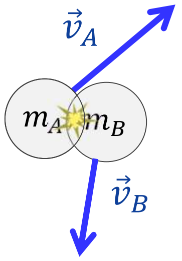
(completely) inelastic impact

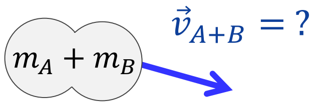
BEFORE:



Momentum:

$$m_A\ \vec{v}_A + m_B\ \vec{v}_B$$

AFTER:



the only unknown, so ...

Momentum:

$$(m_A + m_B)\ \vec{v}_{A+B}$$

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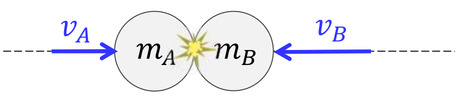
(completely) elastic impact: 1D case

BEFORE:

signed scalar



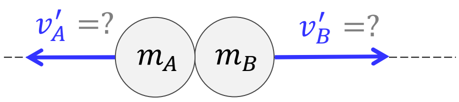
momentum:

$$m_A\ v_A + m_B\ v_B$$

energy:

$$\frac{1}{2} m_A\ {v_A}^2 + \frac{1}{2} m_B {v_B}^2$$

AFTER:



momentum:

$$m_A\ v'_A + m_B\ v'_B$$

energy:

$$\frac{1}{2} m_A\ {v'_A}^2 + \frac{1}{2} m_B {v'_B}^2$$

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(completely) elastic impact: 1D case

new velocities are defined by the impulses:

$$v'_A = v_A + \frac{i_A}{m_A} \qquad v'_B = v_B + \frac{i_B}{m_B}$$

signed scalars

momentum conservation:

$$i_B = -i_A$$

(it's just the 3rd law of dynamics)

energy conservation:

$$\cancel{\tfrac{1}{2}} m_A v_A^2 + \cancel{\tfrac{1}{2}} m_B v_B^2 = \cancel{\tfrac{1}{2}} m_A v_A'^2 + \cancel{\tfrac{1}{2}} m_B v_B'^2$$

$$\Longrightarrow m_A v_A^2 + m_B v_B^2 = m_A \left(v_A + \frac{i_A}{m_A} \right)^2 + m_B \left(v_B + \frac{i_B}{m_B} \right)^2$$

$$\Longrightarrow \cancel{m_A v_A^2} + \cancel{m_B v_B^2} = \cancel{m_A v_A^2} + \frac{i_A^2}{m_A} + 2\, v_A\, i_A + \cancel{m_B v_B^2} + \frac{i_B^2}{m_B} + 2\, v_B\, i_B$$

$$\Longrightarrow 0 = \frac{i_A^2}{m_A} + 2\, v_A\, i_A + \frac{i_B^2}{m_B} + 2\, v_B\, i_B$$

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(completely) elastic impact: 1D case

substituting:

$$\frac{i_A^2}{m_A} + 2\, v_A\, i_A + \frac{i_A^2}{m_B} - 2\, v_B\, i_A = 0$$

$$i_A^2 \frac{m_A + m_B}{m_A\, m_B} + i_A\, 2(v_A - v_B) = 0$$

$$i_A \left(i_A \frac{m_A + m_B}{m_A\, m_B} + 2(v_A - v_B) \right) = 0$$

solution 1

$$i_A = i_B = 0$$

before the impact

solution 2

$$i_A = \frac{2\, m_A\, m_B}{m_A + m_B} (v_B - v_A)$$

after the impact

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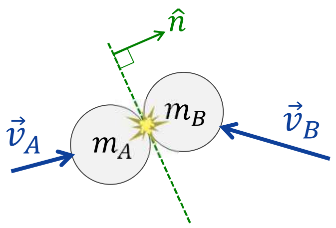
(completely) elastic impact: 3D case

BEFORE:



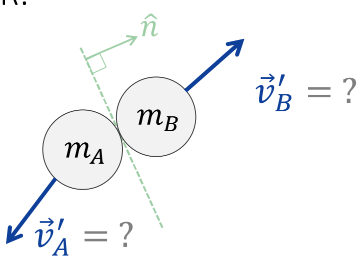
momentum:

$$m_A \vec{v}_A + m_B \vec{v}_B$$

energy:

$$\frac{1}{2} m_A \|\vec{v}_A\|^2 + \frac{1}{2} m_B \|\vec{v}_B\|^2$$

AFTER:



momentum:

$$m_A \vec{v}'_A + m_B \vec{v}'_B$$

energy:

$$\frac{1}{2} m_A \|\vec{v}'_A\|^2 + \frac{1}{2} m_B \|\vec{v}'_B\|^2$$

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(completely) elastic impact: 3D case

- Additional assumption:
  - ∃ impact plane, with normal $\hat{n}$ (we need this info!)
    - o, in 2D: impact line
  - impulses must be orthogonal to this plane $\vec{\imath}_{A,B} = i_{A,B} \hat{n}$ (vector impulses = scalar impulses, pos. or neg. (the unkonwns) times $\hat{n}$)
- To solve the impact
  - find scalar velocities $v_{A,B}$ as the component of vector velocities $\vec{v}_{A,B}$ along $\hat{n}$: $v_{A,B} = \vec{v}_{A,B} \cdot \hat{n}$
  - find scalar impulses $i_{A,B}$ (use the 1D case)
  - find vector impulses $\vec{\imath}_{A,B} = i_{A,B} \hat{n}$
  - apply them to vector velocities

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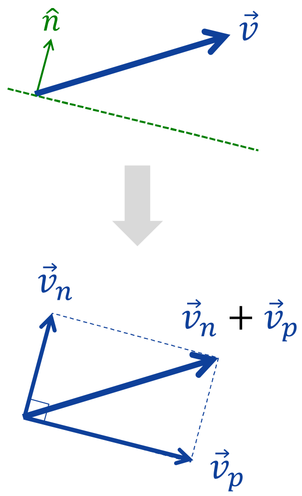
Remember this geometric subproblem?

- Given velocity vector $\vec{v}$ and the impact plane normal $\hat{n}$, split $\vec{v}$ in the vector sum $\vec{v} = \vec{v}_n + \vec{v}_p$ with
  - $\vec{v}_n$ orthogonal to the plane (= parallel to $\hat{n}$)
  - $\vec{v}_p$ parallel to the plane (= orthogonal to $\hat{n}$)
- Solution in 3 steps:
  - (1) $s_n \leftarrow \vec{v} \cdot \hat{n}$ (signed) speed s_n : a scalar
  - (2) $\vec{v}_n \leftarrow s_n \, \hat{n}$ velocity $\vec{v}_n$: a vector
  - (3) $\vec{v}_p \leftarrow \vec{v} - \vec{v}_n$
- Useful because:
  - only $\vec{v}_n$ is affected by elastic impacts with plane
  - only $\vec{v}_p$ is affected by frictions with plane (e.g.: dump it!)
  - s_n is used to *solve* elastic impacts (use 1D case)



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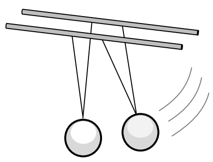
Special case: (exercise: verify!) Equal masses

- Completely elastic case (1D):
  - the two velocities just *swap*
- Completely elastic case (3D):
  - The two velocity components orthogonal to the impact plane *swap*
- Completely inelastic case (3D):
  - the new velocity of both particles is the (vector) average of their pre-impact velocities



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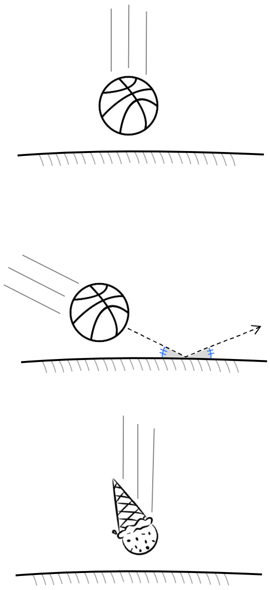
Special case: (exercise: verify!) one-way collision (A is static)

$m_A \longrightarrow \infty$ & $\vec{v}_A = 0$

- Completely elastic case (1D):
  - v_b just *flips*
- Completely elastic case (3D):
  - The component of v_B orthogonal to impact plane just *flips*
- Completely inelastic case (3D):
  - B stops dead ($\vec{v}'_B = 0$)



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Notes on impacts between *rigid bodies*

that is, considering angular velocities too

- We only have seen impacts between *particles*
  - i.e., we disregarded angular velocities
  - when rigid bodies are implicitly implemented as particles + distance constraints, this is all we need to do!
  - Effect of elastic / inelastic impacts on angular velocities will be an (approximated) emerging behavior
- Impacts between *explicit* rigid bodies require to *explicitly* compute the two post-impact angular velocities too
- Different math, stemming form the same principles:
  - Angular momentum: it is *always* preserved, no matter what
  - *Anelastic impact*: post-impact angular velocities must also match
  - *Elastic impact*: kinetic rotational energy must also be preserved
  - *Bounciness* ∈ $[0,1]$: interpolate angular velocities of the above

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