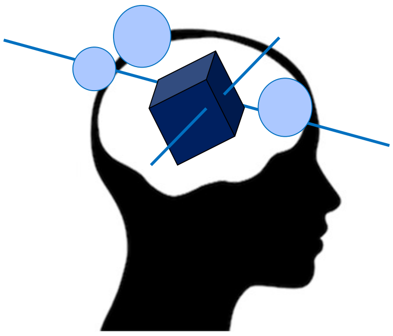
3D VideoGames - UniMi

Points, Vectors, Versors (recap)

Marco Tarini



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Course Plan

- lec. 1: Introduction
- lec. 2: Mathematics for 3D Games
- lec. 3: Scene Graph
- lec. 4: Game 3D Physics +
- lec. 5: Game Particle Systems
- lec. 6: Game 3D Models
- lec. 7: Game Textures
- lec. 9: Game Materials
- lec. 8: Game 3D Animations
- lec. 10: Networking for 3D Games
- lec. 11: 3D Audio for 3D Games
- lec. 12: Rendering Techniques for 3D Games
- lec. 13: Artificial Intelligence for 3D Games

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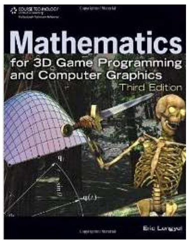
Suggested reading



Mathematics for 3D Game Progr. and C.G. (3rd ed)
Eric Lengyel
Chapters 2, 3, 4

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Point, Vectors, Versors and Spatial Transformation

They are the basic data-type of 3D Games

- In the computation, for all modules
  - rendering engine
  - physics engine
  - AI
  - 3D sound
  - …
- In the data structures of all 3D Assets
  - Meshes, animations, etc

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Point, Vectors, Versors

	represents:	example:	imagine it as...
Point	A position A location	Where a character is The center of a sphere	a small floating dot :-D
Vector	A displacement The difference between 2 points. The vector that connects them.	The velocity of a thrown knife The gravity acceleration How to reach the head of a character from its neck	a small arrow :-D *(length is relevant)*
Versor aka unit vector (as length = 1) aka normal aka direction aka normalized vector	A direction A facing	The view direction of a character The facing of a plane in 3D (i.e. its "normal") The direction of a line, or a ray A rotation axis	the same :-D *(its length is irrelevant)*

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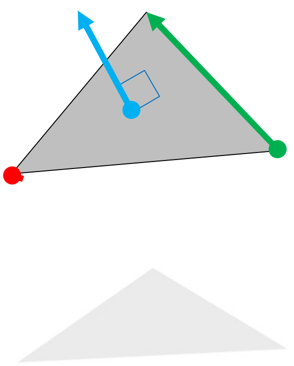
Points, Vectors, Versors
...on a 3D floating tirangle

Examples of...

- point:
  - one vertex of the triangle
- vector:
  - one side of the triangle
- versor:
  - the «normal» of the triangle



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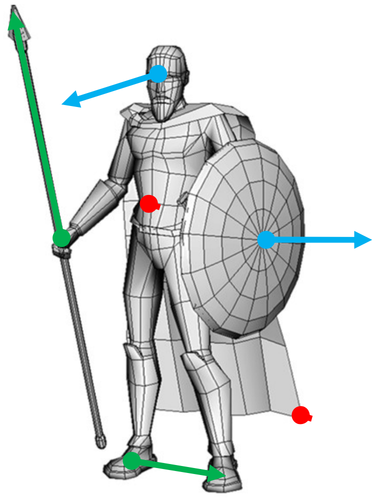
Points, Vectors, Versors
...in a character

Examples of...

- points
- vectors
- versors



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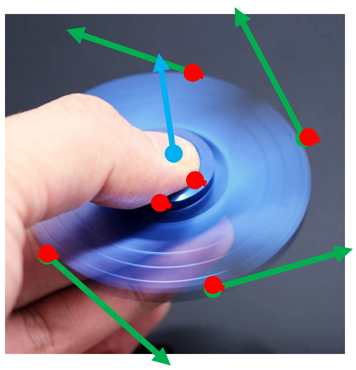
Points, Vectors, Versors
...in a spinner

Examples of...

- points
- vectors
- versors



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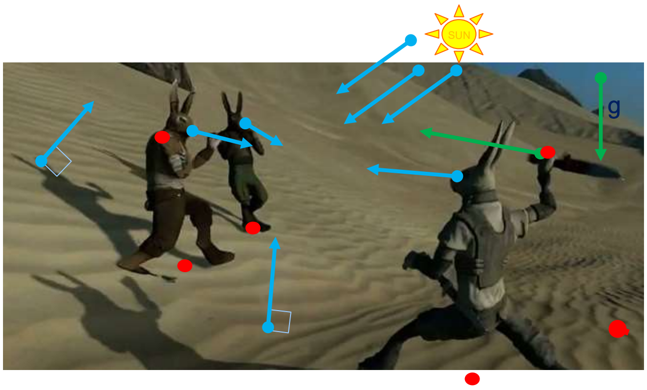
Points, Vectors, Versors
...in this screenshot



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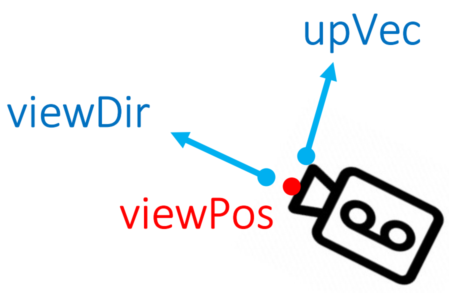
Stuff = Points + Vectors + Versors



Description of a camera
(its so called «extrinsic parameteres»)

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Stuff = Points + Vectors + Versors

description of
a (directional) sound emitter

description of
a (directional) microphone

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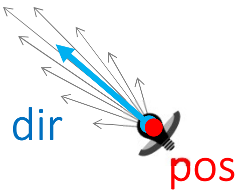
Stuff = Points + Vectors + Versors



description of a spotlight

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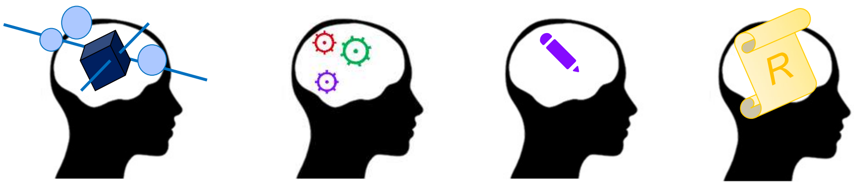
Point, vector, versor *algebra*

make sure to understand
each operation in 3 different ways



intuitive
know how
to imagine it
spatially,
in 3D

operational
know how to
compute
from data

syntactic
know how to
write down,
(in paper
or code)

plus rules:
know how
to manipulate
equations

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The *algebra* of points, vectors, versors (and scalars)

- also, familiarize with the way the operations behave, i.e. rules such as

(1) commutativity? associativity? (of each operation)
(2) distributivity? (between pairs of operations)
(3) inverse operation? identity element? absorbing element?

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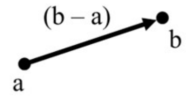
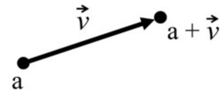
Point and vector algebra (summary 1/7)

- Difference:
  point – point = vector



- Addition:
  point + vector = point



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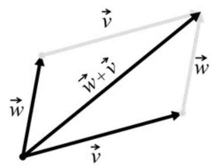
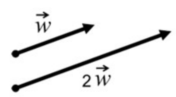
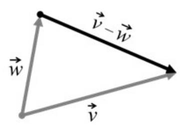
Point and vector algebra (summary 2/7)

- Linear operations for vectors
  - addition (vector + vector = vector)
  - product with a scalar (scaling) (vector * scalar = vector)
  - therefore: interpolation
    $\text{mix}(\overrightarrow{v_0}, \overrightarrow{v_1}, t) = (1-t)\,\overrightarrow{v_0} + t\,\overrightarrow{v_1}$
  - therefore: opposite (flip verse) (how to: multiply by – 1)
  - therefore: difference (vector – vector = vector)







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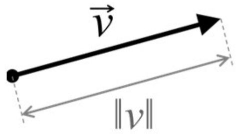
Point and vector algebra (summary 3/7)



- Norm (for vectors)
  - aka length / magnitude / Euclidean norm / 2-norm
  - distance between points:
    length of vector (a – b) = distance between a and b
  - Rules: triangle inequality:

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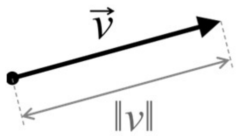
Point and vector algebra (summary 4/7)



- Normalization
  - Input: a vector. Result: a versor
  - how to: scale the vector by (1.0 / length)

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Point and vector algebra (summary 5/7)

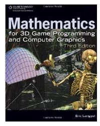
- Dot product (or inner product)
  - Output: a scalar
  - Alternative notations:

$$\vec{v} \cdot \vec{w}$$

$$< \vec{v} , \vec{w} >$$

$$(v^{\mathrm{T}} \, \vec{w})$$

See...



Section 2.2

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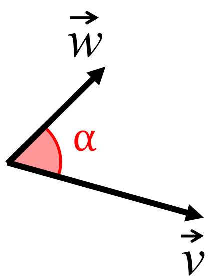
Point and vector algebra (summary 5/7)

- Dot product (or inner product)



$$\vec{v} \cdot \vec{w} \quad = \quad \|\vec{v}\| \cdot \|\vec{w}\| \cdot \; \cos(\alpha)$$

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Point and vector algebra (summary 5/7)

- Dot product, useful to:
  - dot is zero: vectors are orthogonal (or, either vector is degenerate)
  - positive dot: acute angle
  - negative dot: obtuse angle

  } valid with both vectors & versors

  - versor dot vector: extension of vector along direction
  - versor dot versor: cosine of angle
  - versor dot versor: also, a similarity measure (in -1 +1)
  - any vector dot itself: its squared length

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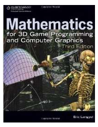
Point and vector algebra Products: additional reading

To be continued!

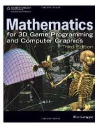
Products between vectors and/or versors



Section 2.2

- Dot product (or inner product)
  - Output: a scalar

NEXT LECTURE!



Section 2.3

- Cross product (or vector product)
  - Output: a vector (note: not a versor)

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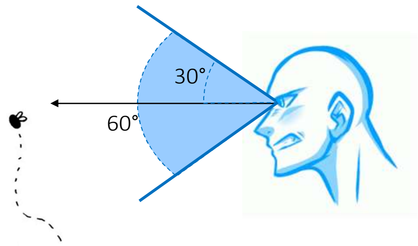
3D videogames

Points, Vectors, Versors: mini task and exercises Part I

Marco Tarini



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Points, Vectors, Versors: mini problems

- The following are examples of spatial problem problems that need to be solved in 3D games
  - They can be solved simply using point/vector/versor algebra
  - Many game engines libraries implement functions for many of them
- General schema for finding a solution:
  - identify input and output (and their types)
  - maybe draw a schema
  - write the equations driven by the drawing, (using your spatial understanding of the operations): e.g. what is orthogonal to what?
  - identify the unknowns
  - manipulate the equations according to the rules to extract extract the unknowns
  - if coding: everything is ready to code it!

For some of them, the solution will be given in full here. In other, only a trace of the solution is given

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Point to point distance (trivial)

"When the player in position p is closer than k to a powerup in pos q, then the powerup is collected"

- Data: p, q points, k scalar
- Test: $\|\mathrm{p}-\mathrm{q}\| < k$
- Optimize vers: $\|\mathrm{p}-\mathrm{q}\|^2 < k^2$
- Pseudo-code example:

```
vec3 p,q;
scalar k;
if ( dot(p-q,p-q) < k*k ) then /*collect*/
```

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Ray-Plane intersection Ver0

"I shoot a laser from p in direction $\hat{\mathrm{d}}$ toward a plane which contains points q and has normal $\hat{\mathrm{n}}$.
Which point q do I hit?"

- Trace:
  - Define q as a point on the laser (see Ray-Sphere inters.)
  - Define q as a point on the plane
    (hint: the vector connecting it to any other point on the plane is orthogonal to $\vec{\mathrm{n}}$)
  - Combine the two equations into one
  - Extract the only incognita

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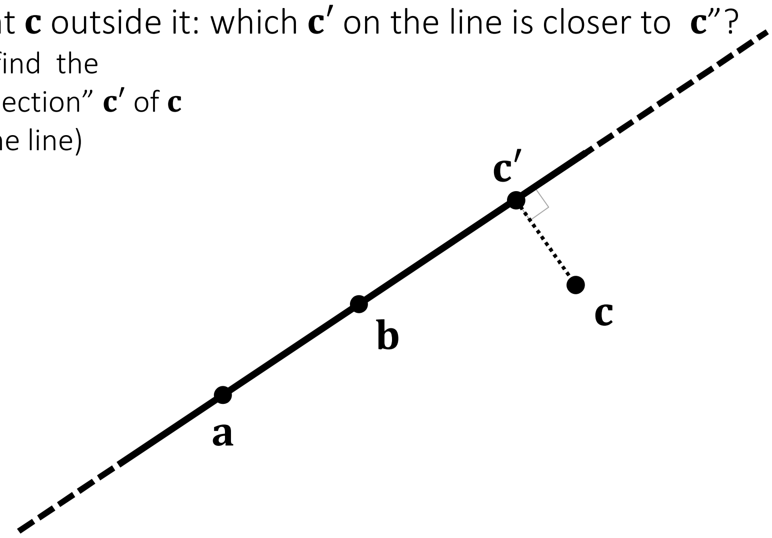
Projection of a point on a line

"Consider a line passing through points $\mathbf{a}$ and $\mathbf{b}$ and a point $\mathbf{c}$ outside it: which $\mathbf{c}'$ on the line is closer to $\mathbf{c}$"?
(i.e. find the "projection" $\mathbf{c}'$ of $\mathbf{c}$ on the line)



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Sphere-sphere intersection (trivial)

"Given two spheres with center in $\mathbf{c}_0$ and $\mathbf{c}_1$ and radii r_0 and r_1: do they intersect? Do they touch?"

- Hint:
  - remember that working with *squared norms* is more efficient (and more accurate) than working with vector norms

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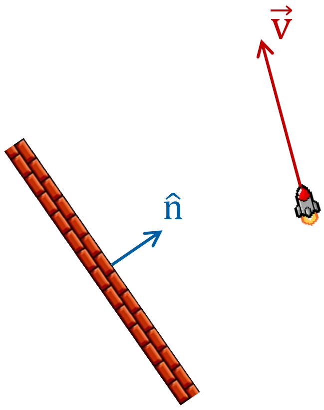
The missile and the wall (trivial)

"A missile is moving at constant velocity $\vec{v}$ (meter per sec), in the general proximity of a large (infinite) wall with normal $\hat{n}$. After some time t (sec), how much closer to (or farther from) the wall is it?"



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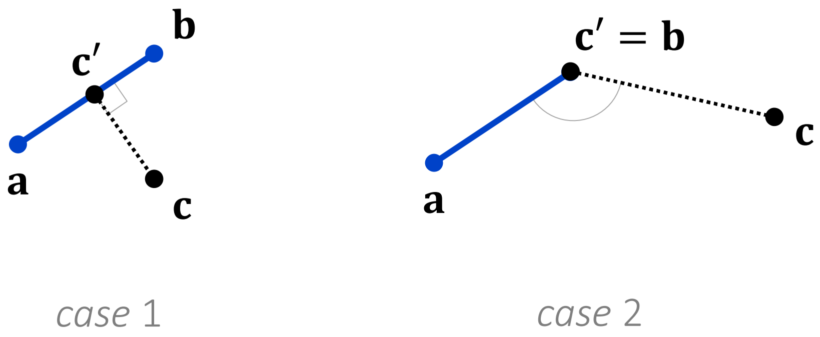
Projection of a point on a segment

"Which $\mathbf{c}'$ point on a segment connecting point $\mathbf{a}$ and $\mathbf{b}$ is closer to a third point $\mathbf{c}$"?



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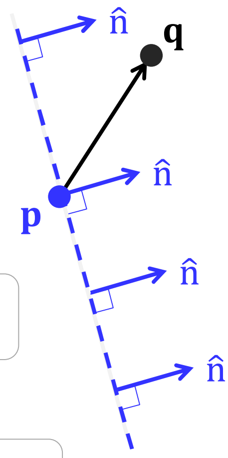
Plane VS Point test

- Input: a point $\mathbf{q}$
  and a plane given by:
  - its normal: $\hat{\mathrm{n}}$
  - a point on it at random: $\mathbf{p}$
- *Q*: on which side of the plane is $\mathbf{q}$?
- *A*: it's the sign of
  $\hat{\mathrm{n}} \cdot (\mathbf{q} - \mathbf{p}) =$
  $\hat{\mathrm{n}} \cdot \mathbf{q} - \vec{n} \cdot \mathbf{p} =$
  $\hat{\mathrm{n}} \cdot \mathbf{q} + k =$
  $(n_x, n_y, n_z, k) \cdot (q_x, q_y, q_z, 1)$

$k = -\vec{n} \cdot \mathbf{p}$
(minus distance of plane from origin)

a 4D vector representing the plane:
a more convenient representation for a plane



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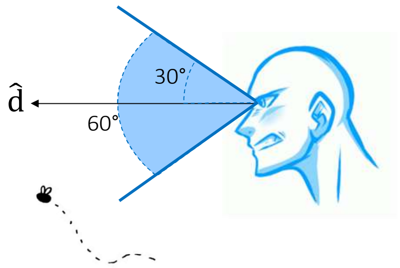
Vision cones

"A guard has eyes in position q
and looks in direction $\hat{\mathrm{d}}$.
Does it spot a fly in
position p , if his cone
of vision is 60° wide?"

- Hypotheses: no occlusions



- Trace:
  - For angles α, β in 0..90°: $\alpha < \beta \;\; \leftrightarrow \cos(\alpha) > \cos(\beta)$
  - Find cosine of angle between view direction and the vector connecting q to p
  - Determine if this cosine is > $\cos(60°/2)$

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