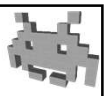
Course Plan



lec. 1: Introduction
lec. 2: Mathematics for 3D Games
lec. 3: Scene Graph
lec. 4: Game 3D Physics +
lec. 5: Game Particle Systems
lec. 6: Game 3D Models
lec. 7: Game Textures
lec. 9: Game Materials
lec. 8: Game 3D Animations
lec. 10: Networking for 3D Games
lec. 11: 3D Audio for 3D Games
lec. 12: Rendering Techniques for 3D Games
lec. 13: Artificial Intelligence for 3D Games

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A transformation class (example) 2/4
Methods to apply them

```
class Transform {
  // fields:
  float s;    // scaling/size
  Rotation r; // rotation/orientation
  Vector3 t;  // translation/position

  // methods:
  Vector3 apply_to_point( Vector3 p ){
      return r.apply_to( s * p ) + t;
  }
  Vector3 apply_to_vector( Vector3 v ){
      return r.apply_to( s * v ); // no traslation
  }
  Vector3 apply_to_versor( Vector3 n ){
      return r.apply_to( n ); // no transl or scaling!
  }
}
```

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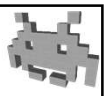
A transformation class (example) 3/4 Composition, inversion, interpolation



- Methods to composite, invert, interpolate

```
class Transform {
  // fields:
  …
  // methods:
  Vec3 apply_to_point( Vec3 p );
  Vec3 apply_to_vector( Vec3 v );
  Vec3 apply_to_versor( Vec3 d );

  Transform composite_with( Transform t );
  Transform inverse();
  Transform interpolate_with( Transform t , float k );
}
```

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A transformation class (example) 3/4 interpolate (aka «blend», «mix», «in-between» …)

Just interpolate the three components

```
class Transform {
  // fields:
  float s;    // uniform scale
  Rotation r; // rotation
  Vec3 t;     // translation

  Transform mix_with( Transform b , float k ){
      Transform result;
      result.s = this.s * k + b.s * (1-k);
      result.r = this.r.mix_with( b.r , k );
      result.t = this.t * k + b.t * (1-k);
      return result;
  }

}
```

black-box for now

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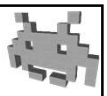
Popular pick for game engines (Unity, Unreal…)



- Rot + Transl. + Non-Uniform scaling
  - how-to: scaling is a vector (not a scalar)
  - you get:
    an unnamed subset of affine transforms.
  - not closed to combination - ☹ ugly!
    - that's why scale of a cumulated transf. is read-only, and approximate
  - but, non-uniform scaling deemed too useful to pass
    - just remember to avoid them if possible
    - e.g. act on 3D models on import – easier, sounder
    - scaling is applied *before* rot and transl. (i.e. «in local space»)
    - if you do use them, apply them early
      i.e. in the leaves of the scene graph tree– see later

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Code example: inverse

It's not enough to invert the 3 components!

```
class Transform {
  // fields:
  float s;      //
  Rotation r;   //
  Vec3 t;       // translation

  Transform inverse(){
      Transform res;
      res.s = 1.0f / this.s;
      res.r = this.r.inverse();
      res.t = -this.t ;



      return res;
  }

}
```

next lecture: we will see inside this class

WRONG!

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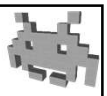
Code example: inverse



```
class Transform {
  // fields:
  float s;     // uniform scale
  Rotation r; // rotation
  Vec3 t;      // translation

  Transform inverse(){
      Transform res;
      res.s = 1.0f / this.s;
      res.r = this.r.inverse();
      res.t = -this.t ;

      res.t = res.r.apply( res.t*res.s );

      return res;
  }

}
```

FIXED!
Takes in account the fixed order (1st scale, then rotate, then translate)

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Inverting a transformation: the math

- Current transform: $f(\mathrm{p}) = \mathbf{R}(s\,\mathrm{p}) + \vec{\mathbf{t}}$

the rotation, the scaling, the translation

- Inverse transform: $f^{-1}(\mathrm{p}) = \mathbf{R}'(s'\,\mathrm{p}) + \vec{\mathbf{t}}'$

the *new* rotation, the *new* scaling, the *new* translation

- Important: the order of operations is the same!
- The problem: how to find $\mathbf{R}'$, s', $\vec{\mathbf{t}}'$ such that
  if $f(\mathrm{p}) = \mathrm{q}$
  then $f^{-1}(\mathrm{q}) = \mathrm{p}$

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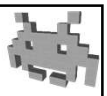
$f(\mathrm{p}) = \mathrm{q}$

$f^{-1}(\mathrm{q}) = \mathrm{p}$



$$\mathrm{q} = \mathbf{R}(s\,\mathrm{p}) + \vec{\mathbf{t}}$$

$\leftrightarrow$

$$\mathrm{q} - \vec{\mathbf{t}} = \mathbf{R}(s\,\mathrm{p})$$

$\leftrightarrow$ apply inverse rot on each side

$$\mathbf{R}^{-1}(\mathrm{q} - \vec{\mathbf{t}}) = s\,\mathrm{p}$$

$\leftrightarrow$

$$\mathbf{R}^{-1}(\mathrm{q} - \vec{\mathbf{t}})/s = \mathrm{p}$$

$\leftrightarrow$ rot ations are are linear funct

$$\mathbf{R}^{-1}(\mathrm{q})/s + \mathbf{R}^{-1}(-\vec{\mathbf{t}})/s = \mathrm{p}$$

$\leftrightarrow$ not valid for non-uniform scalings!

$$\mathbf{R}^{-1}\left(\frac{1}{s}\mathrm{q}\right) + \mathbf{R}^{-1}(-\vec{\mathbf{t}})/s = \mathrm{p}$$

the *new* rotation — the *new* scaling — the *new* translation (a vector)

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Code example: composite (or, cumulate)

It's not enough to compose the 3 components

```
class Transform {
  // fields:
  float s;
  Rotation r;
  Vec3 t; // translation

  Transform cumulateWith( Transform b ){
      Transform result;
      result.s = this.s * b.s;
      result.r = this.r.cumulateWith( b.r );
      result.t = this.t + b.t;
      return result;
  }

}
```

WRONG!

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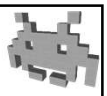
Code example: composite (or, cumulate)



```
class Transform {
  // fields:
  float s;
  Rotation r;
  Vec3 t; // translation

  Transform cumulateWith( Transform b ){
      Transform result;
      result.s = this.s * b.s;
      result.r = this.r.cumulateWith( b.r );
      result.t = b.r.apply( this.t * b.s) + b.t;
      return result;
  }

}
```

correct

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Compositing transformations: the math

stored transformation components

- Input transforms: $f_A(\mathrm{p}) = \mathbf{R}_a(s_a\ \mathrm{p}) + \vec{\mathbf{t}}_a$
  $f_B(\mathrm{p}) = \mathbf{R}_b(s_b\ \mathrm{p}) + \vec{\mathbf{t}}_b$
- Composite transf: $f_{AB}(\mathrm{p}) = \mathbf{R}'(s'\,\mathrm{p}) + \vec{\mathbf{t}}'$

output transformation components

- Observe: f_{AB} still uses *one* scaling, rotation, & transl., to be applied in the *same* order
- The problem: how to find $\mathbf{R}'$, s', $\vec{\mathbf{t}}'$ such that $f_{AB}(\mathrm{p}) = f_A(f_B(\mathrm{p}))$ (first B, then A)

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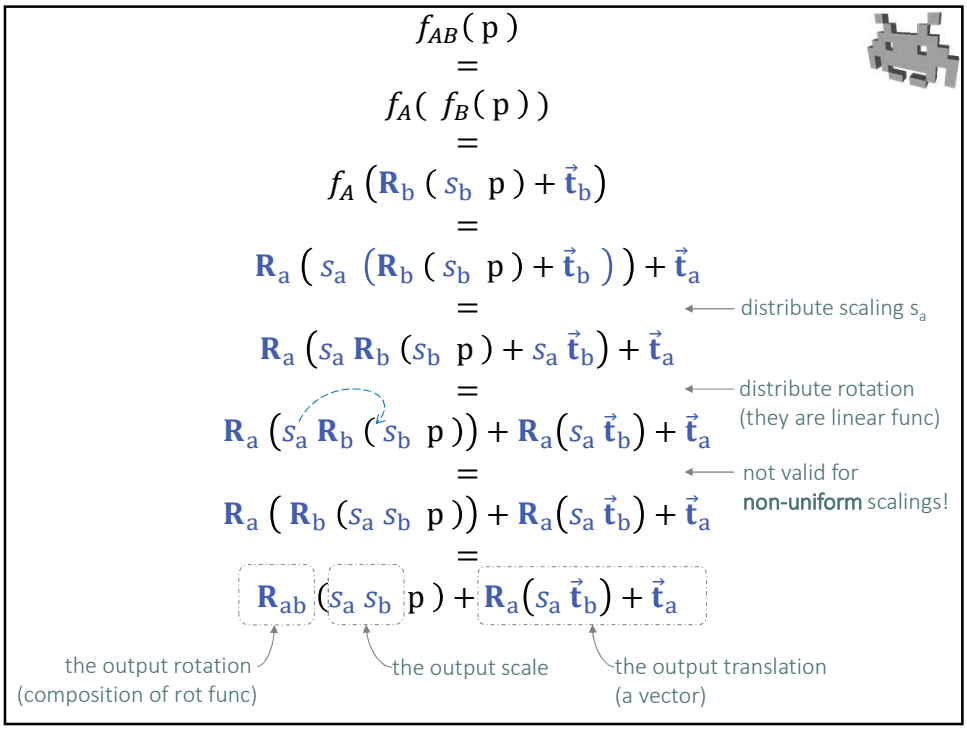

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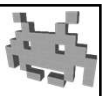
Example: in unity



Class `Transform` with methods:

- `Vector3 TransformPoint(Vector3 pos)`
- `Vector3 TransformVector(Vector3 vec)`
- `Vector3 TransformDirection(Vector3 dir)`

No "invert" method but:

- `Vector3 InverseTransformPoint(Vector3 pos)`
- `Vector3 InverseTransformVector(Vector3 vec)`
- `Vector3 InverseTransformDirection(Vector3 dir)`

Mix: manually mix rotation, scaling, translation components

Cumulation: automatic when needed: see lecture on scene graph

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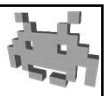
Example: in UNREAL ENGINE



Class `FTransform` with methods:

- `FVector TransformPosition( FVector pos )`
- `FVector TransformVector( FVector vec )`
- `FVector TransformVectorNoScale( FVector dir )`

- `FTransform inverse();`
- `FTransform blend( FTransform a, FTransform b );`
- `void accumulate( FTransform a );`

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In conclusion

- if my 3D transformation is represented as
  - a scaling (optional), plus
  - a rotation, plus
  - a translation
- then I can easily / efficiently
  - store it
  - apply it (to points, vectors & versors)
  - composite it (with another transformation)
  - invert it
  - interpolate it (with another transformation)
- ...as long as I can do so with rotations !

the subject of next lecture

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