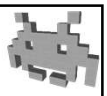
Course Plan



lec. 1: **Introduction**
lec. 2: **Mathematics** for 3D Games
lec. 3: **Scene Graph**
lec. 4: Game **3D Physics**
lec. 5: Game **Particle Systems**
lec. 6: Game **3D Models**
lec. 7: Game **Textures**
lec. 9: Game **Materials**
lec. 8: Game **3D Animations**
lec. 10: **3D Audio** for 3D Games
lec. 11: **Networking** for 3D Games
lec. 12: **Artificial Intelligence** for 3D Games
lec. 13: **Rendering Techniques** for 3D Games

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Forces (continued): control forces

- Example: the player pressing the forward button
  ⇒ a forward force is applied to their avatar
  - no physical justification
  - "Don't ask questions, physics engine"
- According to many:
  it's better when that's not done too much
  - the more physically justified the forces, the better
  - for example: does the car accelerate…
    because a torque is applied to its two traction wheels VS
    because a force is applied to its body
  - van be harder to control
  - see also: gameplay VS cosmetics, control VS realism, emerging behaviors

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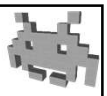
Forces



- Remember all forces acting on a particle add up! (vector summatory)
- The resulting force is what counts

one step

$$\vec{f} \leftarrow fun(\mathbf{p}, \dots)$$
$$\vec{a} \leftarrow \vec{f}/m$$
$$\mathbf{p} \leftarrow \mathbf{p} + \vec{v} \cdot dt$$
$$\vec{v} \leftarrow \vec{v} + \vec{a} \cdot dt$$

$t = t + dt$

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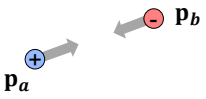
Example of forces: electric forces

- Given two charged particles in $\mathbf{p}_a$ and $\mathbf{p}_b$ with positive or negative charges q_a and q_b

some global constant

$$\overrightarrow{f_a} = \underbrace{\frac{-K\, q_a\, q_b}{\|\mathbf{p}_b - \mathbf{p}_a\|^2}}_{\text{force magnitude (scalar) positive or negative}} \underbrace{\frac{\mathbf{p}_b - \mathbf{p}_a}{\|\mathbf{p}_b - \mathbf{p}_a\|}}_{\text{force direction (versor)}} = \frac{-K\, q_a\, q_b}{\|\mathbf{p}_b - \mathbf{p}_a\|^3} (\mathbf{p}_b - \mathbf{p}_a)$$



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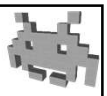
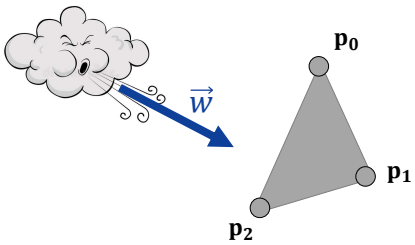
Example of forces: wind pressure



- Wind is a force acting on surfaces
- The larger the exposed surface to the wind, the STRONGER / MORE INTENSE the force
- The more orthogonal the surface to the wind direction, the larger the force
- The stronger the wind pressure $\vec{w}$ (a vector), the larger the force



area vector

$$\vec{f} = \left\| \frac{1}{2} (\mathbf{p_1} - \mathbf{p_0}) \times (\mathbf{p_2} - \mathbf{p_0}) \cdot \vec{w} \right\| \frac{\vec{w}}{\|\vec{w}\|}$$

force magnitude (scalar)

force direction (versor)

(apply 1/3 of $\vec{f}$ on each particle)

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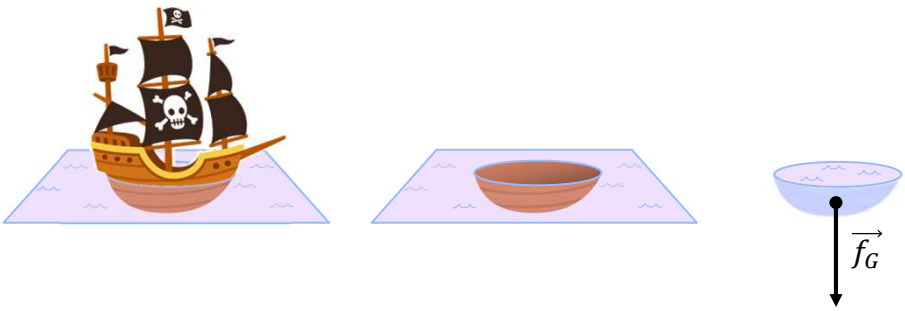
Example of forces: buoyancy

- Opposite of gravity force $\overrightarrow{f_G}$
  - of the submerged part... if it was made of water
  - mass of the submerged part = its volume *times* density of water

or whichever liquid

mass/volume
aka "specific mass"



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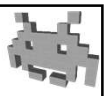
Attrition (or friction) forces



- *Isotropic* friction forces :
  - a force that opposes any motion, regardless of its direction
  - direction: always opposite of current velocity direction
  - magnitude: proportional to the speed
    (= magnitude of velocity vector)
  - note: this force depends on velocity, not positions.
  - models the effect of the medium where the motion happens
    (air, water, thin space…)
  - the denser the medium, the stronger the force
    (water >> air >> thin space)
- Planar friction forces:
  - A force that happens when things slide against each other
  - Always parallel to the contact plane (orthogonal to the normal)
  - Part of the collision response (see next topic!)

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Attrition (or friction) forces:
simulate them with *velocity damping* !

- A useful trick to simulate isotropic friction:
  “velocity damping”
  - simply reduce all velocity vectors by a fixed proportion
  - for example: scale velocity down by 2% per second
    (“drag factor” = 0.02 / sec)
    (that is, scale velocity vectors by a factor 0.98)
  - Why it makes sense:
    Higher speed = more attrition = more loss of speed.
    So, attrition = a “fixed tax” (in %) on velocity.
- For planar friction:
  - Split velocity into parallel / orthogonal parts
  - Apply different Drag factors to each parts

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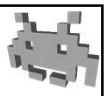
Velocity Damping: how to (as an example)



- Objective: "reduce speed by 1.5% every second"
  - Drag factor: 0.015
- So:
  - After a second: $\vec{v} \leftarrow (1.0 - 0.015)\,\vec{v}$
  - After 2 seconds? $\vec{v} \leftarrow (1.0 - 0.015)^2\,\vec{v}$
  - After k seconds? $\vec{v} \leftarrow (1.0 - 0.015)^k\,\vec{v}$
  - After dt seconds? $\vec{v} \leftarrow (1.0 - 0.015)^{dt}\,\vec{v}$
    - e.g. 1/60 = 0.17 sec
  - Which can be approximated with $\vec{v} \leftarrow (1.0 - 0.015 \cdot dt)\vec{v}$
  - The approximation is good when *this* is small

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Velocity Damping: pseudo-code

```
Vec3 position = …
Vec3 velocity = …

void initState(){
   position = …
   velocity = …
}

void physicStep( float dt )
{
   Vec3 acceleration = force( positions ) / mass;
   position += velocity * dt;
   velocity += acceleration * dt;
   velocity *= (1.0 - DRAG * dt);
}

void main(){
  initState();
  while (1) do physicStep( 1.0 / FPS );
}
```

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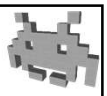
Velocity Damping: notes



- Velocity Damping is useful for robustness
  - Prevents the energy to ever increase
- Limitations:
  - it may exaggerate frictions of, e.g., air, especially in absence of contacts
  - it's a crude approximation: attrition forces are not really *linear* with speed
- In practice:
  - low drag: hardly noticeable (in the short run), increases robustness
  - high drag (e.g. 2% per sec): everything feels like to be moving in molasses (ita: *melassa*); everything quickly grinds to a halt
  - super high drag: (e.g. >25% per sec) basically, no inertia anymore. May be useful to converge to (local) minimal energy states: your simulator is basically a solver for statics, not dynamics

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Continuity of pos and vel

- In real Newtonian physics the state (pos and vel) can only change *continuously*
  - No sudden jump!
- In practice, sometimes is useful to artificially break continuity in the simulations
- Discontinuous changes:
  - for positions: "teleports"
  - for velocity: "impulses"
  - In the real world, those variations can well be consequences of forces, but these forces are not modelled as such, in our system

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Dynamics displacements VS kinematic

a **discontinuous** change of state (position)

…
$\mathrm{p} = \mathrm{p} + \vec{\mathrm{v}} \cdot dt$
…

aka dynamic displacements

Justified by physics

…
$\mathrm{p} = \mathrm{p} + d\mathrm{p}$
…

aka Kinematic displacements

Just "teleportation"

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Impulses VS Forces

a **discontinuous** change of state (velocity)

…
$\vec{v} = \vec{v} + \left(\vec{f} / m\right) \cdot dt$
…

- Forces (continuous)
  - Continuous application
  - every frame

…
$\vec{v} = \vec{v} + \left(\vec{i} / m\right)$
…

- Impulses
  - Infinitesimal time
  - una tantum

they model very intense but short forces (such as impacts)

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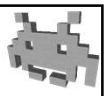
Impulses VS Forces



- Force :
  - it determines an acceleration
  - acc determines a (continuous!) change of vel
  - it's sustained over meaningful periods of time
- Impulse :
  - a (discontinuous!) change of vel
  - in reality: just a force with
    - very large magnitude
    - very short application times
  - we model it as a force "pre-multiplied" with its application time
  - it's useful to:
    - model phenomena with a time scale << *dt*
      e.g. a tennis ball rebounding against a tennis racket
    - control the simulation (direct change of velocity)

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Impulses VS Forces

- what does *truly* happen when it bounces off the ground?

- very strong forces (but not infinite)
- applied for a very short time (but not instantaneous)
- see *collision response* later for details
  about the impulse based approximations

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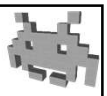
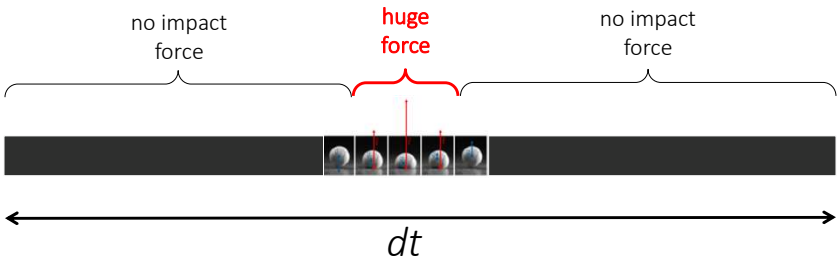
Impulses VS Forces



- what does *truly* happen when it bounces off the ground?



- This can only be modelled as an *impulse*, not a force
- *See also collision response, later*

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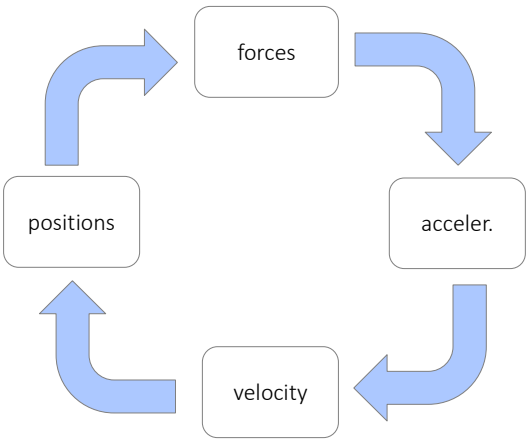
Next: better integration methods for (Newtonian) dynamics



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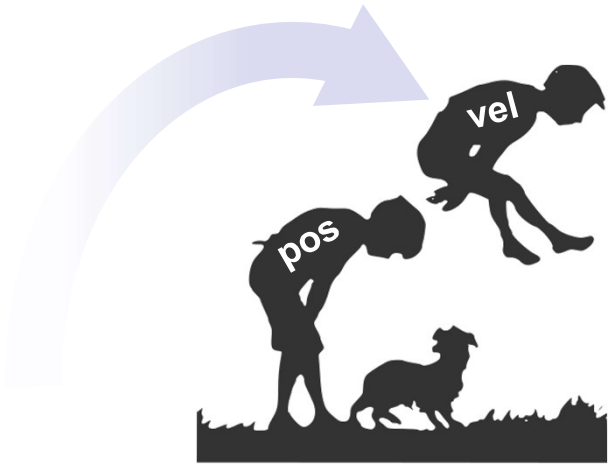
Leapfrog Integration Method



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Leapfrog Integration Method

- Basic Idea:
  store positions at time $k \cdot dt$
  (that is, at t = 0, 1dt, 2dt, 3dt…)
  but store velocities at time $k \cdot dt + \frac{1}{2}\, dt$
  (that is, at t = 0.5dt, 1.5dt, 2.5dt, 3.5dt…)

- Equivalent to use a summatory of
  the areas of trapezoids,
  (having base dt and height $v(t)$)
  not rectangles, to compute the integral

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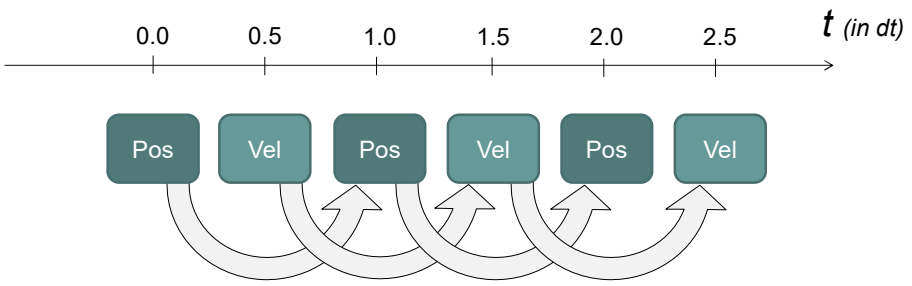
Leapfrog Integration Method



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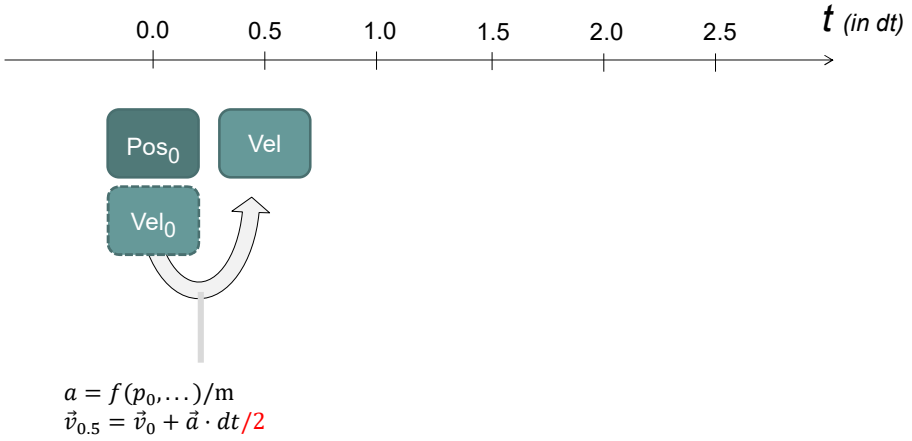
Leapfrog Integration first step



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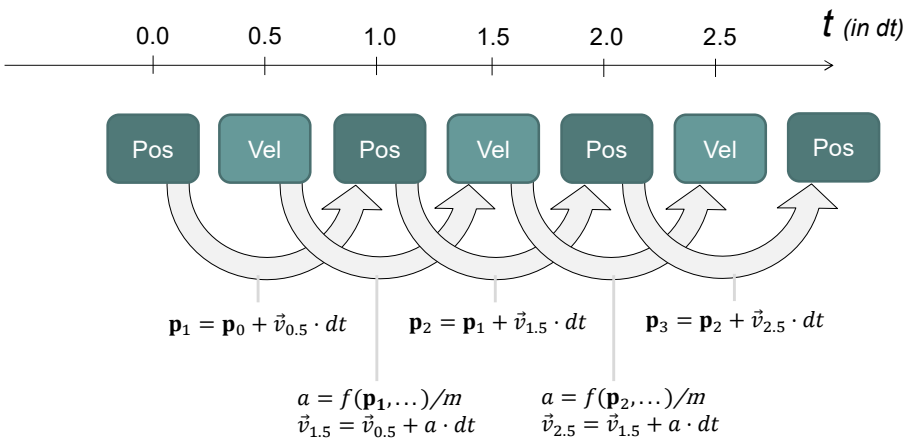
Leapfrog Integration



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Leapfrog method: pros and cons

- Same cost as Euler – and basically same code
  - Velocity stored in status = velocity "half a *dt* ago" (and after updating it: "half a frame in the future")
  - Only real difference: the initialization of velocities
- Better theorical accuracy, for the same *dt*
  - better asymptotic behavior: it's a "second order" system instead of first!
  - cumulated error: proportional to dt^2 instead of dt
  - error per frame: proportional to dt^3 instead of dt^2
- Bonus: fully reversible!
  - in theory only. Beware of numerical errors.
- But: requires fixed *dt* during all the simulation
  - Otherwise, updates of vel required in all particles

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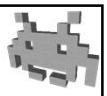
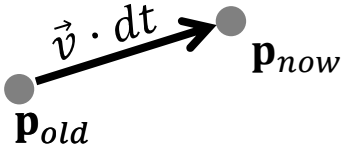
Verlet integration method



- Idea: remove velocity from state
  Instead, store previous position
- Velocity is now implicit
- It's defined by:
  - current pos $\mathbf{p}_{now}$
  - last pos $\mathbf{p}_{old}$
    which we need to record



$$\mathbf{p}_{now} = \mathbf{p}_{old} + \vec{v} \cdot dt$$ ← Euler & variants

$$\Longrightarrow$$

$$\vec{v} = (\mathbf{p}_{now} - \mathbf{p}_{old})/dt$$ ← Verlet

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Verlet integration method: (modifying Euler integration…)

init state

$$\mathbf{p}_{now} = \dots$$
$$\mathbf{p}_{old} = \dots$$

one step

$$\vec{f} = funct(\mathbf{p}_{now}, \dots)$$
$$\vec{a} = \vec{f}/m$$
$$\vec{v} = (\mathbf{p}_{now} - \mathbf{p}_{old})/dt$$
$$\vec{v} = \vec{v} + \vec{a} \cdot dt$$
$$\mathbf{p}_{\text{next}} = \mathbf{p}_{now} + \vec{v} \cdot dt$$

expanding this…

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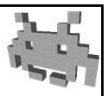
Verlet integration method (modifying Euler integration…)



init state

$\mathbf{p}_{now} \leftarrow \cdots$
$\mathbf{p}_{old} \leftarrow \cdots$

one step

$\vec{f} \leftarrow funct(\mathbf{p}_{now})$
$\vec{a} \leftarrow \vec{f}/m$
$\mathbf{p}_{next} \leftarrow 2\mathbf{p}_{now} - \mathbf{p}_{old} + \vec{a} \cdot dt^2$

$\mathbf{p}_{old} \leftarrow \mathbf{p}_{now}$
$\mathbf{p}_{now} \leftarrow \mathbf{p}_{next}$

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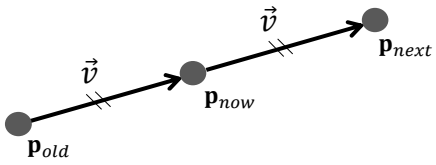
Verlet integration method: geometric interpretation



$\mathbf{p}_{next} = 2 \cdot \mathbf{p}_{now} - 1 \cdot \mathbf{p}_{old}$

$\mathbf{p}_{next}$ can be written as an extrapolation of $\mathbf{p}_{now}$, $\mathbf{p}_{old}$:

$\mathbf{p}_{next} = mix(\mathbf{p}_{old}, \mathbf{p}_{now}, 2)$

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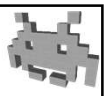
Verlet: characteristics



- Velocity is kept implicit
  - but that doesn't save RAM:
    we need to store previous position instead
  - (a point instead of a vector: same memory)
- Good efficiency / accuracy ratio
  - accumulated error: order of dt^2 (second order method)
- Extra bonus: reversibility
  - it's possible to go backward* in *t* and
    reach the initial state from any state
    * (just swap p_now and p_old)
  - only in theory… careful with implementation details

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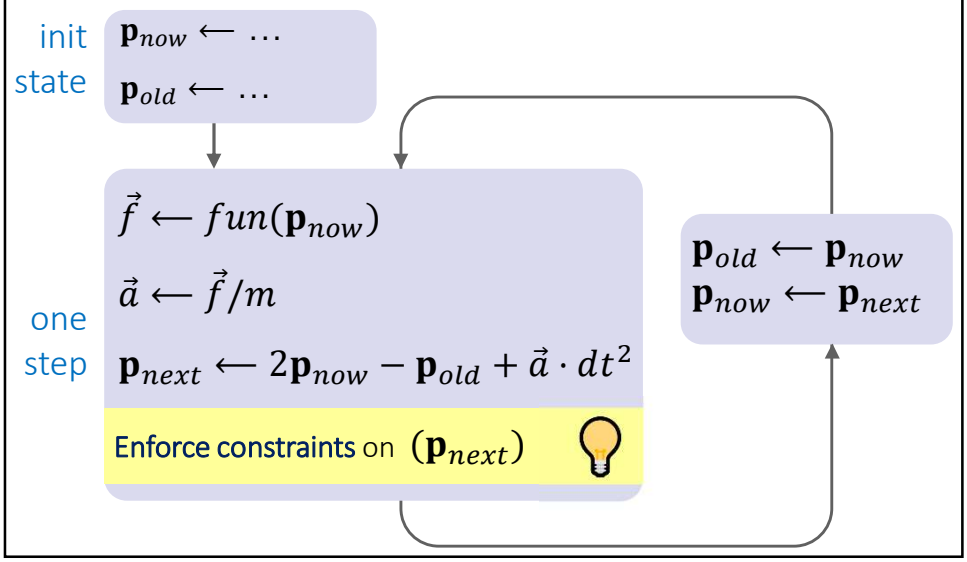
Verlet integration + "Position Based Dynamics" (PBD)



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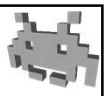
Position Based Dynamics (PDB)



- A positional constraint is an equality/inequality (a formula with '=' '>' '<' etc.) involving the *positions* of particles.
  - Useful, for example, to model consistency conditions
  - Like "*solid objects don't compenetrate each other*", or "*steel bars won't become shorter or longer than they are*"
  - We will see many examples

We enforce (impose) positional constraint directly by displacing the *positions* of particles

- Thanks to Verlet: this displacement automatically causes some appropriate update of the velocity!
- it's not necessarily correct, but it's plausible and robust

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Example of a positional constraint

«I want all particles to stay above ground (that is, their y must never be negative) »

Enforce this constraint: trivial!

```
for (each particle i)
{
  if (p[i].y < 0) p[i].y = 0;
}
```

Imposing constraints like this one is a first part of collision response. For re-bounces, impulses must still be added (see collisions).

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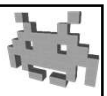
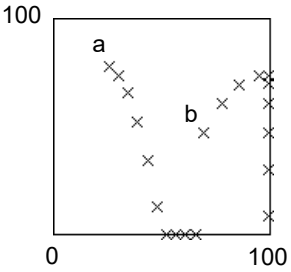
Example of a positional constraint (here, in 2D physics)



«I want particles to stay inside a 2D box [0 – 100] x [0 – 100] *»*



Enforce this constraint: simple clamp!

```
for (each particle i)
{
  p[i].x = clamp( p[i].x, 0, 100 );
  p[i].y = clamp( p[i].y, 0, 100 );
}
```

Imposing constraints like this one is a first part of collision response. For re-bounces, impulses must still be added (see collisions).

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Verlet + Position Based Dynamics. Advantages

- flexibility: different constraints can be used to model many different phenomena
  - Useful constraints are straightforward to define
  - They are easy to impose (they involve only few particles)
  - They can be used to model many possible phenomena
  - See following slides for examples
- robustness : plausibility is ensured by *explicitly* enforcing the conditions we want to see
  - For example: a ball won't ever be seen outside the box containing it – and it will also recover from mistakes
- No forces / impulses are needed to enforce any such consistency conditions

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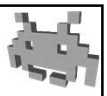
Verlet: *caveats*
(see next slides for solutions)



- ⚠ it assumes a constant *dt* (time-step duration)
  - if *dt* varies: corrections are needed! (how?)
- ⚠ Q: how to act on velocity (which is now implicit)?
  - for example, how to apply impulses ?
  - A: change $\mathbf{p}_{old}$ instead (how?)
- ⚠ Q: how to act of positions w/o impacting velocity?
  - for example, to apply teleports / kinematic motions ?
  - A: change both $\mathbf{p}_{new}$ and $\mathbf{p}_{old}$ (how?)
- ⚠ Q: how to apply velocity damps?

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Changing the value of dt in Verlet
(whenever it's not constant, for any reason)

Problem:

if dt now changes to a new dt'

then, all $\mathbf{p}_{old}$ must be updated to some $\mathbf{p}'_{old}$

Find $\mathbf{p}'_{old}$:

$$\vec{v} = (\mathbf{p}_{now} - \mathbf{p}_{old})/dt$$
$$\vec{v} = (\mathbf{p}_{now} - \mathbf{p}'_{old})/dt'$$

current velocity $\vec{v}$ and position $\mathbf{p}_{now}$ must *not* change, so we can only change $\mathbf{p}_{old}$

$$\Longrightarrow$$

$$\mathbf{p}'_{old} = \mathbf{p}_{now} \cdot (dt - dt')/dt + \mathbf{p}_{old} \cdot dt'/dt$$

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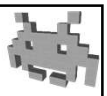
Velocity damping in Verlet



implicit

- Velocity at next frame: $\vec{v} = (\mathbf{p}_{next} - \mathbf{p}_{now})/dt$
- We want to multiply $\vec{v}$ a factor c_{DAMP}
  - before applying accelerations

e.g. 0.98 obtained as $(1\text{-}dt \cdot c_{\text{DRAG}})$

- We can do that using a more general formula for $\mathbf{p}_{next}$

$$\mathbf{p}_{next} = 2 \cdot \mathbf{p}_{now} - 1 \cdot \mathbf{p}_{old} + dt^2 \cdot \vec{a}$$

$$\mathbf{p}_{next} = (1 + c_{\text{DAMP}}) \cdot \mathbf{p}_{now} - c_{\text{damp}} \cdot \mathbf{p}_{old} + dt^2 \cdot \vec{a}$$

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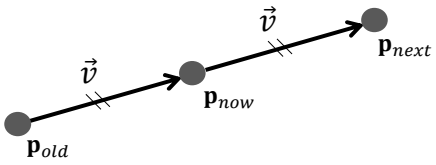
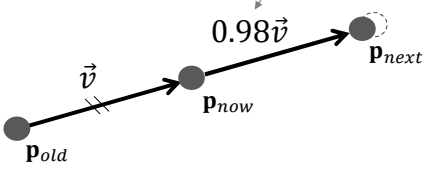
Velocity damping in Verlet (geometric interpretation)

a bit shorter





$$\mathbf{p}_{next} = 2 \cdot \mathbf{p}_{now} - 1 \cdot \mathbf{p}_{old}$$

Equivalently,
$\mathbf{p}_{next}$ is an extrapolation of $\mathbf{p}_{now}$, $\mathbf{p}_{old}$:

$$\mathbf{p}_{next} = mix(\mathbf{p}_{old}, \mathbf{p}_{now}, 2)$$

$$\mathbf{p}_{next} = 1.98 \cdot \mathbf{p}_{now} - 0.98 \cdot \mathbf{p}_{old}$$

Equivalently,
$\mathbf{p}_{next}$ is a different extrapolation of $\mathbf{p}_{now}$, $\mathbf{p}_{old}$:

$$\mathbf{p}_{next} = mix(\mathbf{p}_{old}, \mathbf{p}_{now}, 1.98)$$

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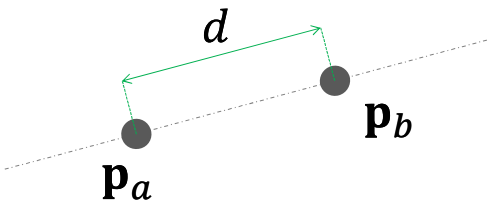
Example of positional constraint: equidistance constraint

*«Particles **a** and **b** must stay at a fixed distance **d** »*



I want that... $\|\mathbf{p}_a - \mathbf{p}_b\| = d$

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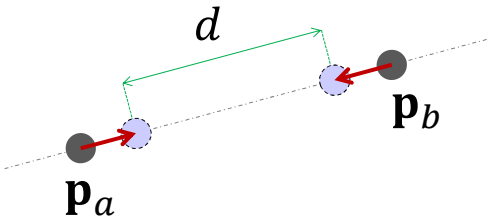
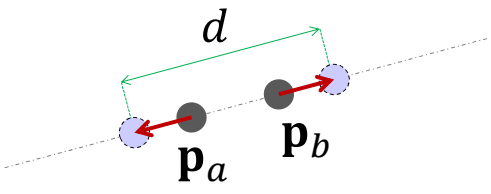
Enforce equidistance constraints
(assuming equal masses for now)

if $\|\mathbf{p}_a - \mathbf{p}_b\| > d$



if $\|\mathbf{p}_a - \mathbf{p}_b\| < d$



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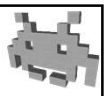
Enforce equidistance constraints: pseudo code



```
Vector3 pa, pb; // curr positions of a,b
float d;        // distance (to enforce)

Vector3 v = pa - pb;
float currDist = v.length;

v /= currDist;  // normalization of v

float delta = currDist - d ;

pa += ( 0.5 * delta) * v;
pb -= ( 0.5 * delta) * v;
```

we move each particle *half the way*
(it makes sense, but see later for why exactly)

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Compare: equidistance constraints vs. springs

- Similar
  - they both mean:
    these 2 particles "want to be" at *this* distance (not more, not less)

some constant scalar parameter D

- but different
  - equidistance constraint:
    - applied during constraint enforcement
    - directly affects positions
    - models a rigid rod between the two particles
      - of a given length
    - sometimes called a "HARD" constraint
  - spring:
    - applied during force evaluation step
    - affects forces, therefore accelerations
    - models a deformable spring between the two particles
      - of a given length
    - sometimes called a "SOFT" constraint
- A physic engine can use both at the same time!

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How to enforce a positional constraiont?
(see next lecture for the complete anwser)

When enforcing constraints…

- If a constraint is valid, no problem.
- If it's not, change particle positions so that it does.
  - Problems: there's usually many possible ways to do

⚠ Which one to pick?

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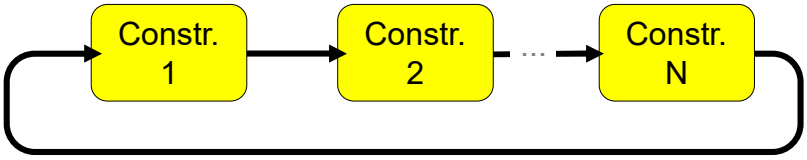
Enforcing a set of constraints

- There are many constraints to impose:
  when you solve one → maybe you break another!
- Simultaneous enforcement: computationally expensive
- Practical & easy solution: just enforce them in cascade
  (similar in concept to Gauss-Seidel solvers):



Repeat until convergence (= max error below threshold)
…but at most for N_{MAX} times! (remember: our simulation is *soft* real-time)

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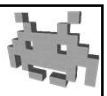
Enforcing a set of constraints one after the other (in cascade)



- The whole loop for imposing the constraints happen in the constraint enforcement phase on one physics step
- Notes about convergence:
  - needed iterations (typically) few: e.g. 1 ~ 10 (efficient!).
  - if convergence not reached within a given number of steps: never mind, next frames will fix it (so it's robust)
  - (it is never reached, if constraints are contradictory)
  - Optimization (to reduce the number of needed iterations): solve the most unsatisfied constraints first

⚠ Problem: it's a sequential approach! ☹

- parallelized versions (similar to Jacobi solvers) are possible
- they have a worse convergence in practice (they require more iterations), but each iteration is faster

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Compounds of particles disguised as rigid bodies

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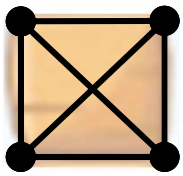
We can combine equidistance constraints to obtain rigid objects!

- Rigid body dynamics as emerging behavior
  - without explicitly keeping track their orientation, angular vel, angular acc., etc.



A box in 2D?
(rigid object)
A configuration of:
- 4 particles
- 6 equidistance constraints

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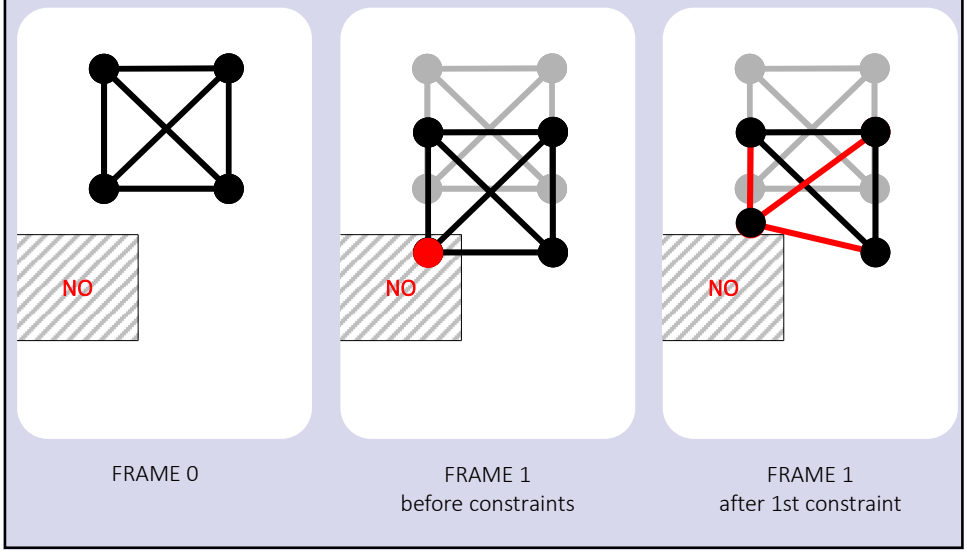
Example



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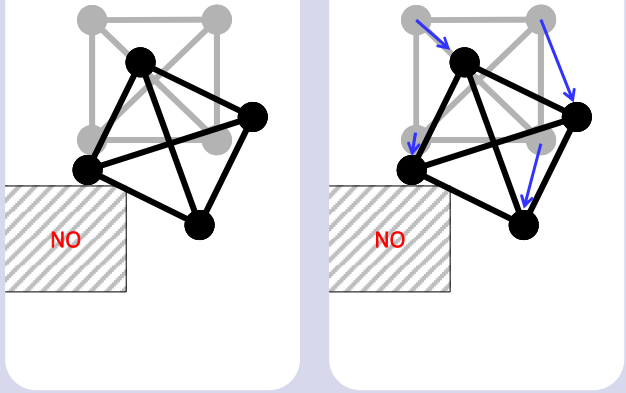
Example



FRAME 1
after all constraints
multiple times

FRAME 1
resulting
(implicit) velocities

In total: the "box",
under gravity + collision
- has rotated
- has gained angular velocity (and will keep rotating by inertia)

…even the system does not (explicitly) handle rigid-bodies (and they rotations, angular velocities, or angular momentum)

(works in 3D as well!)

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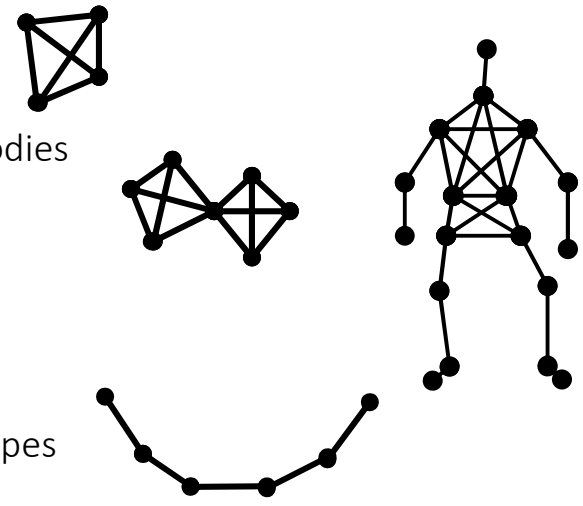
We can combine equidistance constraints to obtain…

- Rigid bodies
- Articulated bodies
- Ragdolls
- Cloth
- Non-elastic ropes
- …and more



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