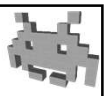
Course Plan



lec. 1: **Introduction**
lec. 2: **Mathematics** for 3D Games
lec. 3: **Scene Graph**
lec. 4: Game **3D Physics**
lec. 5: Game **Particle Systems**
lec. 6: Game **3D Models**
lec. 7: Game **Materials**
lec. 8: Game **Textures**
lec. 9: Game **3D Animations**
lec. 10: **Audio** for 3D Games
lec. 11: **Networking** for 3D Games
lec. 12: **Interactive Agents** for 3D Games
lec. 13: **Rendering Techniques** for 3D Games

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A flashback: Complex Numbers in a nutshell 1/3

- It all starts with a «fantasy» assumption:
  there is an imaginary number i
  such that $i^2 = -1$
  - And for any other purpose, i behaves just like any (non-zero) real number
- Consequences:
  - We now have number of the form $a + b\,i$, *real part* *imaginary part*
    with $a, b \in \mathbb{R}$, called complex numbers (the set is called $\mathbb{C}$)
  - The algebra of complex numbers
    (how to sum, multiply, invert them…)
    is fully determined by the «fantasy» assumption above

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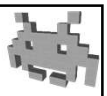
A flashback: Complex Numbers in a nutshell 2/3



- For example, sum:

  real part *imaginary part*

$$(a + b\,i) + (c + d\,i) = (a + c) + (b + d)i$$

- For example, product (remembering $i^2 = -1$):

$$(a + b\,i) * (c + d\,i) = (ac - bd) + (ad + bc)i$$

- For example, inverse (verify it!):

$$(a + b\,i)^{-1} = \frac{(a\ - b\,i)}{a^2 + b^2}$$

the «coniugate» of $(a + b\,i)$

the squared «magnitude» of $(a + b\,i)$

- What is interesting to us is the geometric interpretation of these operations

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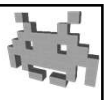
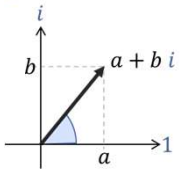
A flashback: Complex Numbers in a nutshell 3/3





- Geometric interpretation:
  - $a + b\,i$ represents the vector/point (a, b)
  - Complex sum = vector sum
  - Conjugate = mirroring with the imaginary axis (vertical)
  - Product = add angles (with real axis), multiply magnitudes
- Therefore:
  - product with a unitary (magnitude = 1) complex number is a 2D rotation around origin
  - A complex number $\mathrm{r} \in \mathbb{C}$ with $\|\mathrm{r}\| = 1$ represents a 2D rot; multiply a vector $(x + y\,i)$ with r means to rotate it

Wouldn't it be nice to have the same for 3D rotations?

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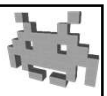
Quaternions



as a table:

×	i	j	k
i	-1	$+k$	$-j$
j	$-k$	-1	$+i$
k	$+j$	$-i$	-1

- New «fantasy» assumption: there are three different "imaginary" numbers i , j , k such that:
  - for any other purpose, i, j, k behave like real numbers

$$\begin{cases} i^2 = k^2 = j^2 = -1 \\ ij = k \ , \quad ji = -k \\ jk = i \ , \quad jk = -i \\ ki = j \ , \quad kj = -j \end{cases}$$

to remember it:

- Consequences:
  - We now have number of the form $a\,i + b\,j + c\,k + d$, with $a, b, c, d \in \mathbb{R}$, called Quaternions (their set is $\mathbb{H}$)
  - The algebra of quaternions (how to sum, multiply, invert them…) is simply determined by the «fantasy» assumptions
  - Again, what is interesting to us is the geometric interpretation…

imaginary parts *real part*

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Quaternions: how to write them (three equivalent ways)

- Algebraic form: $a\,i + b\,j + c\,k + d$
  - often, omitting the zeros, e.g. $i + 2\,k$ is a quaternion
- As vectors of $\mathbb{R}^4$: (a , b , c , d)
- As vector & scalar pairs: ($\vec{v}$, d)

the imaginary part, a 3D vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ *the real part, a scalar*

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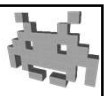
Quaternions: algebra



$\mathbf{q} \in \mathbb{H}$ $\quad\quad$ $\mathbf{q} = ai + bj + ck + d$

- Sum, Scale, Interpolate , etc.:
  - Nothing new, just apply the rules
- Magnitude

consider them as 4D versors

$$\|\mathbf{q}\| = \sqrt{a^2 + b^2 + c^2 + d^2}$$
$$\|\mathbf{q}\|^2 = a^2 + b^2 + c^2 + d^2$$

  - «unitary» if it's 1

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Quaternions: algebra

$\mathbf{q} \in \mathbb{H}$ $\quad\quad$ $\mathbf{q} = ai + bj + ck + d$

- Product: just apply the «fantasy» assumptions
  - Observe: product is not commutative (nor anticommut.)
  - (see later slides to see what the product turns out to be)
- «Conjugate»:
  - like for complex numbers: $\bar{\mathbf{q}} = -ai - bj - ck + d$

flip the imaginary parts

- Inverse: (like for complex numbers) $\mathbf{q}^{-1} = \bar{\mathbf{q}} \,/\, \|\mathbf{q}\|^2$
  - For unitary quat, it's just the conjugated

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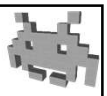
Quaternions: geometric interpretation



- A quaternion $\mathrm{q} = (\vec{\mathrm{v}}, d)$ represents :
  - the 3D point / vector / versor $\vec{\mathrm{v}}$, when $d = 0$
  - a 3D rotation, when q is unit, i.e. $\|\mathrm{q}\|^2 = \|\vec{\mathrm{v}}\|^2 + d^2 = 1$
  - neither, otherwise
- If q is a rotation and p is a point ($\mathrm{q}, \mathrm{p} \in \mathbb{H}$) then the representation of the rotated point / vector / versor is given by:

$$\mathrm{p}' = \mathrm{q} \cdot \mathrm{p} \cdot \bar{\mathrm{q}}$$

quaternion products

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Rest of the lecture (plan)

- Quaternions to-do list
  - Understand *which* rotation is encoded by a quaternion
  - Investigate quaternion multiplication
  - Investigate cumulation, inversion, and interpolation of rotations as quaternions (including shortest path)
  - Examples of rotation as quaternions and their application
- Rotations exercises:
  - "Find the rotation which…"
  - "Convert a rotation representation into another"
- Roto-translation representation
  - Beyond just (rotation , translation) pairs

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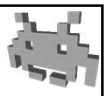
Rotation composition?
Quaternion multiplication!



$p, q_0, q_1 \in \mathbb{H}$

p represents a point
q_0, q_1 represent rotations

p rotated by q1, then rotated by q0

p rotated by q1

$$q_0 \cdot (q_1 \cdot p \cdot \bar{q}_1) \cdot \bar{q}_0$$

product is associative (like for complex numbers)

$$= (q_0 \cdot q_1) \cdot p \cdot (\bar{q}_1 \cdot \bar{q}_0)$$

$\bar{r} \cdot \bar{s} = \overline{s \cdot r}$ (rules of quaternions) (remember: product is not commutative)

$$= (q_0 \cdot q_1) \cdot p \cdot \overline{(q_0 \cdot q_1)}$$

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Rotation inversion?
Quaternion conjugation!

Proof:

$p, q \in \mathbb{H}$

p represents a point
q represents a rotation

p rotated by q, then rotated by $\bar{q}$

p rotated by q

$$\bar{q} \cdot (q \cdot p \cdot \bar{q}) \cdot \bar{\bar{q}}$$

product is associative (like for complex numbers)

$$= (\bar{q} \cdot q) \cdot p \cdot (\bar{q} \cdot \bar{\bar{q}})$$

$$= p$$

Just like with compex numbers:

$$q^{-1} = \frac{\bar{q}}{\|q\|^2} = \bar{q}$$

because q is unitary

that is:

$$\bar{q} \cdot q = \|q\|^2 = 1$$

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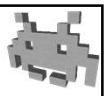
Quaternions: geometric interpretation (complete)



- A quaternion $\mathrm{q} = (\vec{\mathrm{v}}\ ,\ d\ )$ represents :
  - the 3D point / vector / versor $\vec{\mathrm{v}}$, when $d = 0$
  - a 3D rotation, when q is unit, i.e. $\|\mathrm{q}\|^2 = \|\vec{\mathrm{v}}\|^2 + d^2 = 1$
  - neither, otherwise
- If q is a rotation and p is a point ($\mathrm{q}, \mathrm{p} \in \mathbb{H}$) then…
  - $\mathrm{q} \cdot \mathrm{p} \cdot \bar{\mathrm{q}}$ is the rotated point / vector
  - $\bar{\mathrm{q}}$ is the inverse rotation
  - (so, $\bar{\mathrm{q}} \cdot \mathrm{p} \cdot \mathrm{q}$ is point p rotated… in the *other* direction)
  - $\mathrm{q}_0 \cdot \mathrm{q}_1$ is the composited rotation (first q_1 then q_0)

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3D Rotations as Quaternions

- quaternion q representing the 3D rotation of angle α around axis $\hat{\mathrm{a}}$:
  - $\mathrm{q} = \left(\sin\left(\frac{\alpha}{2}\right) \hat{\mathrm{a}}\ ,\ \cos\left(\frac{\alpha}{2}\right) \right)$

  that is
  - $\mathrm{q} = \sin\left(\frac{\alpha}{2}\right) \hat{\mathrm{a}}_x i + \sin\left(\frac{\alpha}{2}\right) \hat{\mathrm{a}}_y j + \sin\left(\frac{\alpha}{2}\right) \hat{\mathrm{a}}_z k + \cos\left(\frac{\alpha}{2}\right)$
- Observe that $\|\mathrm{q}\|^2 = 1$ ← verify

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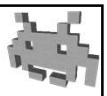
Exercise: are the following quaternions unitary?



- $\mathbf{q}_0 = (0,0,-1,0) = -j$
- $\mathbf{q}_1 = \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right) = 0.5i + 0.5j + 0.5k + 0.5$
- $\mathbf{q}_2 = (1,1,1,1) = i + j + k + 1$

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Quaternions: exercises (solutions in next slides)

- Which quaternion encodes a turnabout?
  - (ita: «*un dietrofront*»: turning 180° around the up vector)
- Apply that quaternion to rotate a point in (x,y,z)
  - Use plain quaternion algebra, and algebraic notation
- Which quaternion encodes the identity rotation?
  - Is it the only one? If not, which other does?
  - Verify by applying it (or them)
- Which quaternion encodes «turn of 90° to your left»?
- Uses your previous *two* answers to find the quaternion encoding «turn 45° to your left», *by using interpolation*
  - Do you need SLERP in this case? Is NLERP enough? Why?
  - Verify that the solution is correct extracting axis & angle

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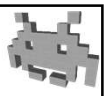
Example: turnabout rotation
(italian: un «dietrofront»)



- Find the quaternion $\mathbf{r}$ representing the rotation by 180° (π radiants) around axis Y
  - $\hat{a} = (0,1,0)$
  - $\alpha = \pi, \sin\left(\frac{\alpha}{2}\right) = 1, \cos\left(\frac{\alpha}{2}\right) = 0$
  - $\mathbf{r} = (1\,\hat{a}, 0) = 0i + 1j + 0k + 0 = j$

imaginary vector — real scalar

- Find the quaternion $\mathbf{q}$ representing point $\begin{pmatrix}2\\3\\4\end{pmatrix}$:
  - $\mathbf{q} = 2i + 3j + 4k$
- Rotate that point with that rotation:
  - $\mathbf{q}' = \mathbf{r}\,\mathbf{q}\,\bar{\mathbf{r}} = j\,(2i + 3j + 4k)(-j) = \ldots$ *(finish me!)*

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3D Rotations as Quaternions: equivalent representations ☹

- Around axis $\hat{a}$ by angle α:

$$q = \left(\sin\left(\frac{\alpha}{2}\right)\hat{a}, \cos\left(\frac{\alpha}{2}\right)\right)$$

- Around axis $-\hat{a}$ by angle $(-\alpha)$ (it's the same rotation!):

$$q' = \left(-\sin\left(\frac{-\alpha}{2}\right)\hat{a}, \cos\left(\frac{-\alpha}{2}\right)\right) = q$$

the **same** quaternion :-)

Nice! But:

360°

- Around axis $\hat{a}$ by angle $(\alpha + 2\pi)$ (again, it's the same rotation!):

$$q'' = \left(\sin\left(\frac{\alpha}{2} + \pi\right)\hat{a}, \cos\left(\frac{\alpha}{2} + \pi\right)\right) =$$

180°

$$= \left(-\sin\left(\frac{\alpha}{2}\right)\hat{a}, -\cos\left(\frac{\alpha}{2}\right)\right) = -q$$

a **different** quaternion :-(

- Conclusion:
  quaternion q and quaternion $-q$ encode the same rotation

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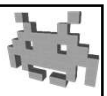
3D Rotations as Quaternions: equivalent representations ☹



Given a quaternion $\mathbf{q}$ representing a rotation:

- Flip its imaginary part (getting $\bar{\mathbf{q}}$): invert rotation
- Flip its real part (getting $-\bar{\mathbf{q}}$): invert rotation
- Flip everything (getting $-\mathbf{q}$): same rotation

Every single rotation is encoded
by two different quaternions: $\mathbf{q}$ and $-\mathbf{q}$.
(the inverse rotation is also encoded
by two different quaternions: $\bar{\mathbf{q}}$ and $-\bar{\mathbf{q}}$)

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Interpolating two quaternions (that represent two rotations)

Works well, but two *caveats*:

⚠ Take the "shortest path" (as usual):
flip 2nd quaternion first, if this makes them closer

- Distance defined as dot product in 4D
(consider quaternions as 4D unit vectors for this)
(remember: dot product between unitary vectors is a measure of similarity!)

⚠ Loss of normality

- Needs re-normalization (NLERP),
- Or SLERP
(again, just consider quaternions as 4D unit vectors)

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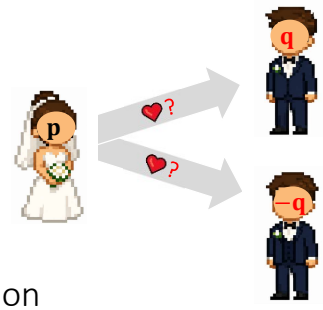
Shortest path interpolation: the case of quaternions



- Let $\mathbf{p}$ and $\mathbf{q}$ be two rotations
- $\mathbf{q}$ and $-\mathbf{q}$ represent the same rotation (as a state).
  - Which one to choose? alike choose alike
- Which one is closer to $\mathbf{p}$?
  - Similarity between $\mathbf{p}$ and $\mathbf{q}$ $= \mathrm{dot}(\mathbf{p}, \mathbf{q})$
  - Similarity between $\mathbf{p}$ and $-\mathbf{q}$ $= \mathrm{dot}(\mathbf{p}, -\mathbf{q}) = -\mathrm{dot}(\mathbf{p}, \mathbf{q})$
- Conclusion:
  - If $\mathrm{dot}(\mathbf{p}, \mathbf{q})$ is positive, interpolate with $\mathbf{q}$
  - Otherwise, interpolate with $-\mathbf{q}$

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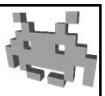
Quaternions, exercise:



Experiment with cumulation rotations

1. Take the quaternion q0 that encodes the 180° rotation around the Y axis (see exercises above)
2. Take the quaternion q1 that encodes the 180° rotation around the X axis (see exercises above)
3. Compute the quaternion q2 that does the two rotations in succession, in that order (using q0 and q1)
4. Which rotation is encoded by q2? Verify with a real 3D object (e.g. a cellphone) that q2 encoded the status that is reached if you rotate by q0 and then by q1

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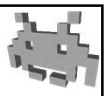
Quaternions, exercise:
Experiment with interpolating rotations



1. Take (again) the quaternion q0 that encodes the 180° rotation around the Y axis
2. Take a quaternion q1 that encodes identity rotation
   - (question: is there only one of them?)
3. Compute the quaternion q2 that interpolates the two quaternions q0 and q1
   - what happens with the shortest path?
   - why do you think that happens?
4. Which rotation is encoded by q2? To help with the answer, the sin and cos for PI/4 radiants (45°) is…

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Quaternion Product

×	a *i*	+	b *j*	+	c *k*	+	d
e *i*		+		+		+	
+							
f *j*		+		+		+	
+							
g *k*		+		+		+	
+							
h		+		+		+	

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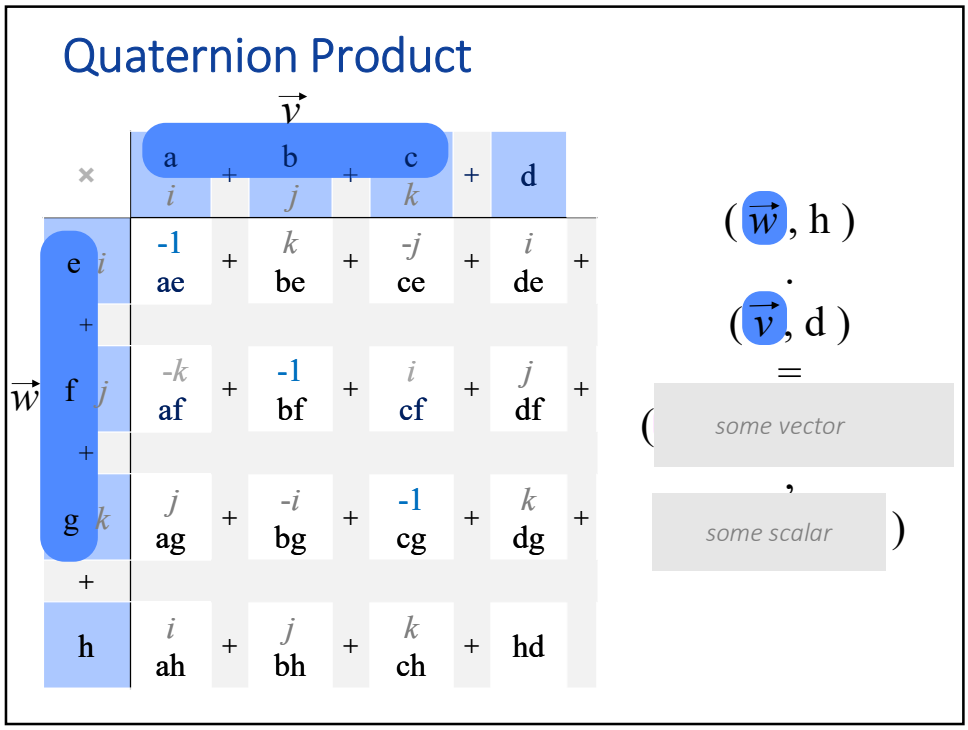
Quaternion Product
$\vec{v}$
a i + b j + c k + d
$\vec{w}$
e i + f j + g k + h
-1 ae + k be + -j ce + i de +
-k af + -1 bf + i cf + j df +
j ag + -i bg + -1 cg + k dg +
i ah + j bh + k ch + hd
($\vec{w}$, h) . ($\vec{v}$, d) =
(some vector , some scalar)

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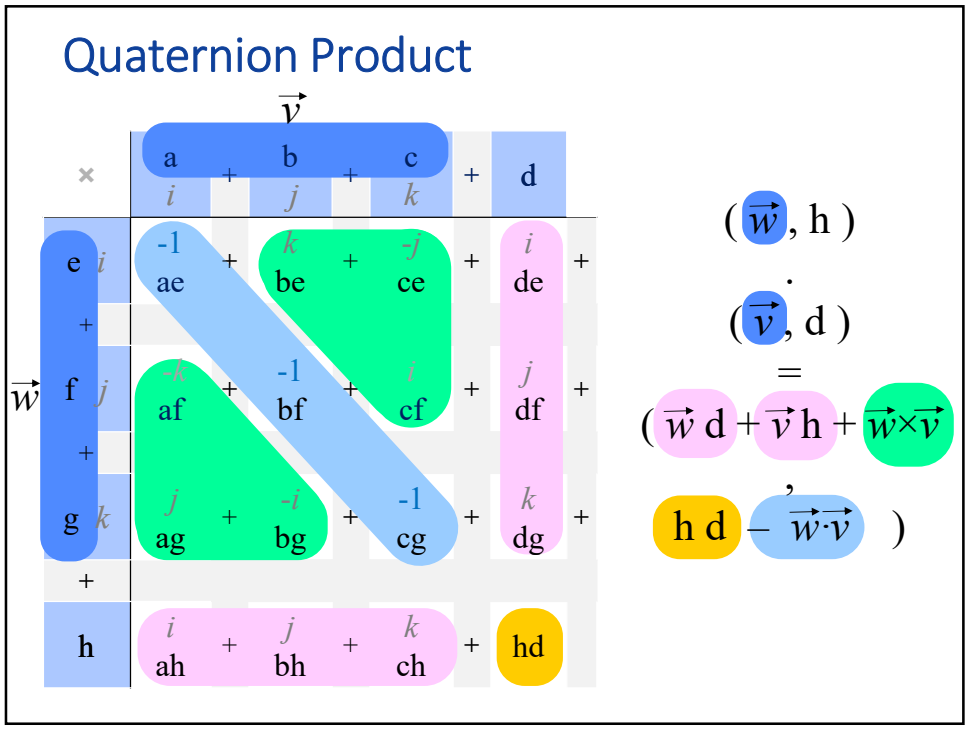
Quaternion Product
$\vec{v}$
a i + b j + c k + d
$\vec{w}$
e i + f j + g k + h
-1 ae + k be + -j ce + i de +
-k af + -1 bf + i cf + j df +
j ag + -i bg + -1 cg + k dg +
i ah + j bh + k ch + hd
($\vec{w}$, h) . ($\vec{v}$, d) =
($\vec{w}$ d + $\vec{v}$ h + $\vec{w}\times\vec{v}$, h d − $\vec{w}\cdot\vec{v}$)

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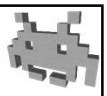
Quaternion multiplication: notes



- The previous slide shows how to compactly write (and compute! and code!) a product between two quaternions that are expressed as pairs (imaginary-vector , real-scalar)
- Quaternion multiplication is the basic operation, used (twice) to apply a rotation to a point/vector/versor, and (once) to cumulate two quaternions
- For example (next slide): let's use this in the formula to rotate to a point/vector/versor with quaternions
  - and let's see what simplifies
- Note: do not confuse the **quaternion product** with the **dot product** of two quaternions seen as 4D vectors

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Applying rotations (as quaternions)

quaternion representing a rotation; quaternion representing 3D point or vector $\vec{v}$; conjugate of $(\vec{w}, a)$

$$(\vec{w}, a)\ (\vec{v}, 0)\ (-\vec{w}, a) =$$

$$= (\vec{w}, a)\ (a\vec{v} - \vec{v} \times \vec{w}, \vec{v} \cdot \vec{w}) =$$

$$= \left(a\,\vec{w} \times \vec{v} - \vec{w} \times \vec{v} \times \vec{w} + (\vec{v} \cdot \vec{w})\vec{w} + a^2\,\vec{v} - a\,\vec{v} \times \vec{w},\ \cancel{a\vec{v} \cdot \vec{w}} - \cancel{a\vec{v} \cdot \vec{w}} + \cancel{\vec{w} \cdot (\vec{v} \times \vec{w})} \right) =$$

why?

$$= (a^2\,\vec{v} + 2a\,\vec{w} \times \vec{v} + (\vec{w} \cdot \vec{v})\,\vec{w} - \vec{w} \times \vec{v} \times \vec{w},\ 0)$$

of course

Optimized!

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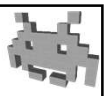
Exercise: quaternion norm as a quaternion product



- As you may remember,
  given a complex number $\mathbf{c} \in \mathbb{C}$, $\mathbf{c} = a + ib$
  its magnitude $\|\mathbf{c}\| = \sqrt{a^2 + b^2}$
  can be expressed as
  $$\|\mathbf{c}\|^2 = \mathbf{c}\,\bar{\mathbf{c}}$$
- Does the same hold for quaternions?
  Given $\mathbf{q} \in \mathbb{H}$:
  $$\|\mathbf{q}\|^2 = \mathbf{q}\,\bar{\mathbf{q}}$$
- Verify, using the multiplication formula we learnt

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Quaternions as rotations: summary

- Compact to store (4 scalars, almost the minimum)
- Trivial to invert (just conjugate)
- Fast to composite (just multiply: $2^{nd} * 1^{st}$)
- Fast to apply (multiply twice, with natural and conjugated)
- Easy to enforce that it stays a rotation (just renormalize)
  - Even after long sequences of cumulations, unlike matrices
- Behaves well under interpolation
  - Just use NLERP – even better with SLERP
  - Remember to take the shortest path (=> flip sign if necessary)
- The favorite representation in 3D games
  - but, other solutions still useful in one context or another

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Recap: representing rotations 1/2

	3x3 Matrix	Euler Angles
Space efficient? (in RAM, GPU, storage…)	★☆☆☆☆ 9 scalars	★★★★★ 3 scalars (even as small int!)
Efficient/easy to: Apply (to points/vectors)	★★★★★ 9 products (3 dot products)	★★☆☆☆ requires trigonometry sin/cos
Efficient/easy to: Invert (produce inverse)	★★★★★ just transpose	★☆☆☆☆
Efficient/easy to: Composite (with another rotation)	★★☆☆☆ Matrix multipl (9 dots) Numerical errors	★☆☆☆☆
Efficient/easy to: Interpolate (with another rotation)	★☆☆☆☆ Introduces shear/scale	★☆☆☆☆ easy to do, unintuitive result (⚠ shortest-path required!)
Intuitive? (e.g. to manually set)	★★★☆☆	★★★★★ roll yaw pitch
Notes…	Free extra shear + scale. Useful to extract local axes.	⚠ GIMBAL LOCK

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Recap: representing rotations 2/2

	axis , angle	(unitary) quaternion
Space efficient? (in RAM, GPU, storage…)	★★★★☆ 4 scalars (or 3) (precision needed)	★★★★☆ 4 scalars (precision needed)
Efficient/easy to: Apply (to points/vectors)	★★☆☆☆ Requires trigonometry	★★★★☆ just 2 quat product (& optimizable)
Efficient/easy to: Invert (produce inverse)	★★★★★ Just flip axis OR angle	★★★★★ super easy flip imaginary or real part
Efficient/easy to: Composite (with another rotation)	★☆☆☆☆	★★★★★ super easy: 1 quat product
Efficient/easy to: Interpolate (with another rotation)	★★★★☆	★★★★★ easy + good result (NLERP or SLERP)
Intuitive? (e.g. to manually set)	★☆☆☆☆ no	★☆☆☆☆ no
Notes…	⚠ *two* representations for each rotation (flip all → no effect) (for different reasons) Require shortest path!	

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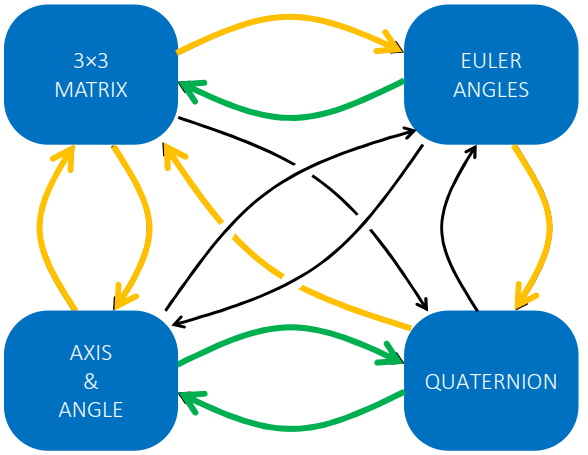
Switching between representations



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What defines a rotation, for you?

« Roll, pitch, and yaw! »

then you are… a pilot, or an astronaut

« X-angle, Y-angle, and Z-angle! »

then you are… a digital artist (like an animator, or a scener)

« An angle! »

then you are… a flatland citizen

« A vector! the dir is the axis the magnitude the angle »

then you are… a physicist

« A 3x3 matrix! the submatrix of a 4x4 transform »

then you are… a computer graphicist, or a Graphics API

« A quaternion! »

then you are… a game developer

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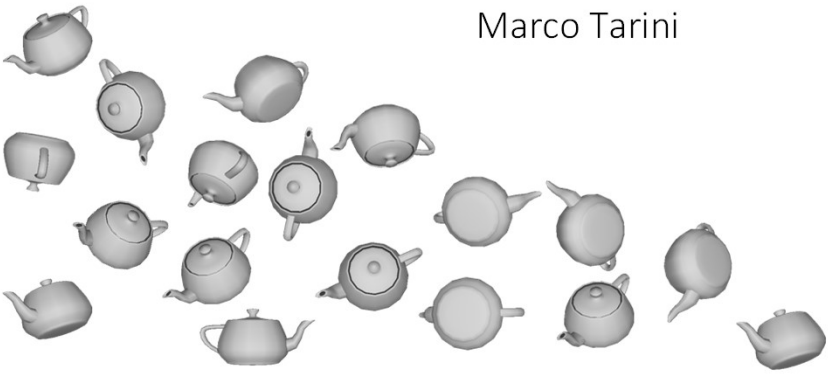
3D Videogames

Transformations in games: a few practical notes about game engines



Marco Tarini

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Notes on transformation in unity

What you see in the GUI

Rotations as Euler angles

- the intuitive choice! (perfect for a GUI)

note: exposed as degrees, not radians
--> even more intuitive

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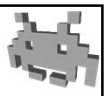
Notes on rotations in Unity internally



Rotations as Quaternion (a class)

- can be initialized (via constructors) as a …
  quaternion, euler angles, axis+angle, or matrix
- can be converted to / accessed as… any of the above
  (thanks to methods, or to "properties" -
  basically setter/getter methods in disguise+
  that gives the illusion for a "quaternion" to be whichever type you think it is
- Support for cumulation (multiplication),
  inversion (conjugation), interpolation (SLERP) …
- Support for all the common problems
  (from-to rotation, etc)

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Notes on Rotations in Unreal Engine

fields: `W X Y Z`

Class **`FQuat`** :

- convert from:
  - axis+angle, matrix4x4, Rotator, euler (vec3) (by constructors)
  - Euler angles (`makeFromEuler` method)
  - From-to vector pairs (`FindBetween` method)
- convert to:
  - `ToAxisAndAngle`, `Euler`, `Rotator`,
  - matrix columns `GetAxis(X|Y|Z)`
  - also, with names: `Get(Forward|Right|Up)Vector`,
- methods: invert with `Inverse`,
  blend with `FastSlerp`
  or `FastSlerpFullPath` (no shortest path)
  apply with `RotateVector` / `UnrotateVector`
  composite with `operator *`

Class `FRotator`
for "nautical" Euler angles:
fields: `Pitch Roll Yaw`

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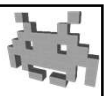
Notes on Rotations in Godot (C# / C++ game engine)



Class **`Basis`**

- a rotation as a 3x3 matix
- Its x,y,z fields (of type Vector3) are the columns
  (that is, the x,y,z axis of local space define in global space)
- Warning: doesn't have to be orthonormal.
  For example, can include scale
- Has a method to enforce back orthonormality

Class **`Quaternion`**

- a rotation as a quaternion
- Includes method for SLERP-ing

The two classes can be converted to/from each other (via constructors)
Both include methods for cumulation (multiplication), application, inversion.
Both include methods for conversion to/from axis-angle.
Euler angles: no explicit support (but naturally you can go from 3 Euler angles to either class by multiplying rotations around x, y, z axis, but no built-in order)

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Notes on rotations in OpenGL (Computer Graphics Library)

- In the «old school» API:
  (and now in many similar libraries)
  - API: glRotate3f
    - takes: angle & axis
  - Internally:
    - matrices
    - jointly as with any other spatial transform
    - separated in MODEL+VIEW+PROJECT transforms

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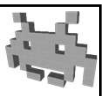
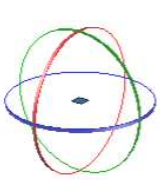
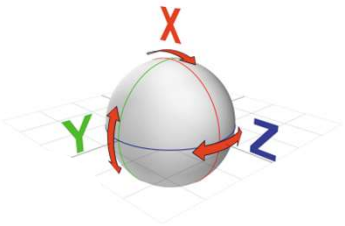
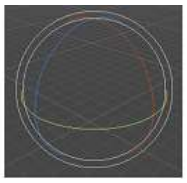
GUI: how do artists author 3D rotations?



- Typical way: rotation gizmo
  - (also: «arcball» or «trackball»)
  - 3 handles to control the three Euler angles
  - or “free”, drag-n-drop mode (trackball metaphor)







convention: Red = X Green = Y Blue = Z

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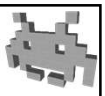
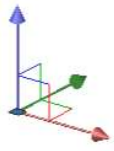
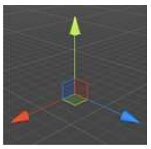
GUI: how do artists author 3D translations?

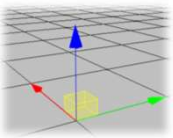


translation gizmo

- handles to traslate along axes or planes





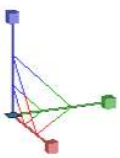


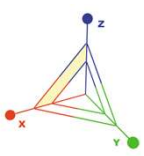
convention:
Red = X
Green = Y
Blue = Z

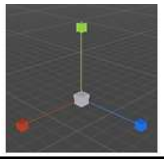
GUI: how to author 3D scalings?

scale gizmo

- 3 handles for anisotropic scalings
  1 handle (middle) for uniform scalings







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