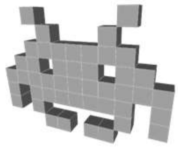
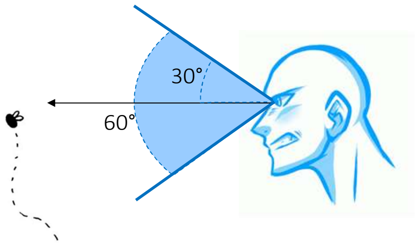
3D videogames

Points, Vectors, Versors: mini task and exercises



Marco Tarini



48

Points, Vectors, Versors: mini task and exercises

- The following exercises (done in the classroom) use the vector
  - In some of them, the solution is given in full
  - In other, only a trace of the solution is given
- General schema for solutions:
  - Identify input and output (and it’s type)
  - Write the equations driven by the intuitive/spatial understanding of the operations,
  - manipulate the equations according to the rules, extract the solution

49

Points, Vectors, Versors: mini task and exercises

- Try to write pseudo-code that solves the proposed problems, using
  - An existing library (GLM, Unity, Unreal) GLSL..
  - Your own hand-made library for points/vectors/versor

50

Point to point distance

"When the player in position p is closer than k to a powerup in pos q, then the powerup is collected"

- Data: p, q points, k versor
- Test: $\|\mathrm{p}-\mathrm{q}\| < k$
- Optimizing: $\|\mathrm{p}-\mathrm{q}\|^2 < k^2$
- Pseudo-code example:

```
vec3 p,q;
scalar k;
if ( dot(p-q,p-q) < k*k ) then /*collect*/
```

51

Orthonormal base completion

"I have a ony two axes $\hat{\mathrm{x}}$ and $\hat{\mathrm{y}}$ of an orthonormal bases, how do I find the third vector $\hat{\mathrm{z}}$?"

- Data: $\hat{\mathrm{x}}$, $\hat{\mathrm{y}}$ versors
- Hypotheses: $\hat{\mathrm{x}}$ and $\hat{\mathrm{y}}$ are already orthogonal
- Variant: $\hat{\mathrm{y}}$ is not exactly orthogonal to $\hat{\mathrm{x}}$, but I want to change it the least to make it orthogonal ($\hat{\mathrm{x}}$ is to be kept constant)
  (see next problem)

52

Vector orthogonalization

"Find a versor $\hat{\mathrm{u}}'$ that is ortogonal to a given $\hat{\mathrm{n}}$ such that it is as similar as possible to a given versor $\hat{\mathrm{u}}$"

Solution: $\hat{\mathrm{u}}' = \hat{n} \times \hat{u} \times \hat{n}$

```
vec3 n,u;
u = cross( cross( n , v ) , n );
```

53

Ray-sphere intersection

"I shoot a laser from p to direction $\hat{\mathrm{d}}$. Do I hit a sphere in position q of radius r ? Where?"

- Data: p, q points, r scalar, $\hat{\mathrm{d}}$ versor
- Trace:
  - Hit-point is s on laser ray:
    $\mathrm{s} = \mathrm{p} + k\,\vec{\mathrm{v}}$, for some unknown scalar $k \geq 0$
  - Hit-point is s on sphere:
    $\|\mathrm{q} - \mathrm{s}\| = r \quad \leftrightarrow \quad (\mathrm{q} - \mathrm{s}) \cdot (\mathrm{q} - \mathrm{s}) = r^2$
  - Combine the two equations (substitute s in second),
    solve for k (it's a 2nd degree equation),
    test that k exists and that it is >0)

54

Shooting a walking target (with a finite speed bullet) 1/2

"I shoot a bullet from p with velocity $\vec{\mathrm{v}}$. At which time the bullet will be the closest to a target currently in position q and moving with velocity $\vec{\mathrm{w}}$? Where will bullet and target be, at that point?"

- Data: p, q points, $\vec{\mathrm{v}}$ and $\vec{\mathrm{w}}$ vectors
- Hypothesis: nothing accelerates (everything keeps moving at a constant speed)

55

Shooting a walking target (with a finite speed bullet) 2/2

Trace

- Position of bullet at time t : $\mathrm{p} + t\,\vec{\mathrm{v}}$
- Position of target at time t : $\mathrm{q} + t\,\vec{\mathrm{w}}$
- Squared distance between the two at time t :

$$\| (\mathrm{p} + t\,\vec{\mathrm{v}}) - (\mathrm{q} + t\,\vec{\mathrm{w}}) \|^2$$
$$=$$
$$\| (\mathrm{p} - \mathrm{q}) + t\,(\vec{\mathrm{v}} - \vec{\mathrm{w}}) \|^2$$

- Work on formulas, derive for *t*, equate derivative to 0, extract *t*

56

Ray-Plane intersection

“I shoot a laser from p in direction $\hat{\mathrm{d}}$ toward a plane which contains points a b c. Which point q do I hit?”

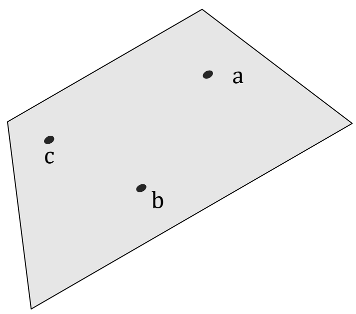
- Hypotheses: a b c are not colinear (not on a line)
- Trace:
  - Find vector $\vec{\mathrm{n}}$ orthogonal to plane, use cross product (magnitude and verse are not important)
  - Define q as point on the laser (see Ray-Sphere inters.)
  - Define q as a point on the plane (hint: the vector connecting it to any other point on the plane is orthogonal to $\vec{\mathrm{n}}$)
  - Combine the two equations into one
  - Wxtract the incognita

57

Sub problem: surface normal

"I have three points on $\mathbf{a}$ $\mathbf{b}$ $\mathbf{c}$ on a plane: find the normal $\hat{\mathbf{n}}$ of this plane (a versor)"

- Trace:
  find any two
  different vectors
  on (i.e. parallel to)
  the plane…

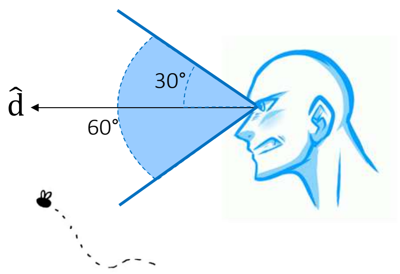


58

Vision cones

"A guard has eyes in position $\mathbf{q}$ and looks in direction $\hat{\mathbf{d}}$. Does it spot a fly in position $\mathbf{p}$, if his cone of vision is 60° wide?"

- Hypotheses: no occlusions



- Trace:
  - For angles α, β in 0..90°: $\alpha < \beta \quad \leftrightarrow \cos(\alpha) > \cos(\beta)$
  - Find cosine of angle between view direction and the vector connecting $\mathbf{q}$ to $\mathbf{p}$
  - Determine if this cosine is > $\cos(60°/2)$

59