## Representations for

roto-translations

- $3 \times 3$ Matrices
- Euler Angles
- Angle + Axis
- Quaternions

- $4 \times 4$ Matrices (or $3 \times 4$ )
- Dual Quaternions

2

Representations for
roto-rotations (notes)

- So far, we assumed that the rotation and translation component of a transformations are stored separatedly
- We have seen reasons why this is convenient
- There are a few representations which store rotation and translation (roto-translations, aka "rigid" transformations) jointly:
- $4 \times 4$ matrices (we have seen the problem with them)
- Dual quaternsions


## Dual Quaternions: their math in a nutshell

- New "fantasy" assumption: there is a $\varepsilon$ such that $\varepsilon \neq 0, \varepsilon^{2}=0$ (for the rest, $\varepsilon$ behaves like any real)
- A dual quaternion: $p+\varepsilon q$, with $p, q \in \mathbb{H}$
- That is, eight scalars
- weights for: $i, j, k, 1, \varepsilon i, \varepsilon j, \varepsilon k, \varepsilon$
- A dual quaternion represents:
dual-quat conjugate
- a point / vector in 3D, when $p=0$ and $\operatorname{Real}(q)=0$
- a roto-translation, when $\|p\|=1$ and $p \cdot q=0$
- To roto-translate a point $a$, with roto-trans $b$ just conjugate their representations $\mathrm{a} \leftarrow \mathrm{b} \cdot \mathrm{a} \cdot \overline{\mathrm{b}}$

4

## Course Plan

lec. 1: Introduction
lec. 2: Mathematics for 3D Games
lec. 3: Scene Graph
lec. 4: Game 3D Physics
lec. 5: Game Particle Systems
lec. 6: Game 3D Models
lec. 7: Game Textures
lec. 8: Game 3D Animations
lec. 9: Game 3D Audio
lec. 10: Networking for 3D Games
lec. 11: Artificial Intelligence for 3D Games
lec. 12: Game 3D Rendering Techniques
5


6

Recap:
3D Spatial Trasforms

- Math functions
- input: point / vector / versor
- output: point / vector / versor

Thus, can be applied to e.g. 3D models (apply it to every vertex position and normal...)

- Typically:
- Scaling + rotation + translation
- They capture:
- Size / scaling up or down
- With deformations (anisotropic) or not (isotropic , uniform)
- Orientation in space / rotation
- Position / movement (translation)

7

Recap: transformation associated to an object in the scene

- Any object associated to a spatial location in the game is given its transformation, which goes
- From:
- local space a.k.a.
- object space a.k.a.
- pre-transform space
- a.k.a. «castle» space / «hero»space / «camera» space / «chainsaw» space / «bazooka» space / etc
9


10


11


## Moving Object: two ways of updating per-object Transforms

- Let $T_{\text {new }}$ be a new transformation to be applied to move object D (w.r.t. its current placement)
- Say: rotation=ide scaling=1 translation=(-2,0,0)
- $\mathrm{T}_{\text {new }}=$ "move two units to the left" (assuming $\mathrm{X}=$ right)
- How to update transformation $\mathrm{T}_{\mathrm{D}}$ ? Two ways:
- $T_{D} \leftarrow T_{D} \cdot T_{\text {new }}=$ object $D$ moves 2 units on its left
- $T_{D} \leftarrow T_{\text {new }} \cdot T_{D} \quad=$ object $D$ moves 2 units on world's left (meaning, i.e., "Westward")

Info: Unity calls this applying the new transformation in local space or in global space respectively both in interface, and in scripts (see parameter relativeTo of Transform.Translate)

Moving Object: two ways of updating per-object Transforms

- Let $T_{\text {new }}$ be a new transformation to be applied to change object D (w.r.t. its current placement)
- Say: rotation = ide scaling $=2$ translation $=(0,0,0)$
- $\mathrm{T}_{\text {new }}=$ "double by $x 2$ " (volume gets $\times 8$ bigger)
- How to update transformation $\mathrm{T}_{\mathrm{D}}$ ? Two ways:
- $T_{D} \leftarrow T_{D} \cdot T_{\text {new }}=$ object $D$ enlarges from its center
- $T_{D} \leftarrow T_{\text {new }} \cdot T_{D}=$ object $D$ enlarges from world's center (i.e. moves away from it too)


## Moving Object: two ways of updating per-object Transforms

- Let $T_{\text {new }}$ be a new transformation to be applied to change object D (w.r.t. its current placement)
- Say: rotation $=j$ scaling $=1$ translation $=(0,0,0)$
- $\mathrm{T}_{\text {new }}=$ "flip by $180^{\circ}$ around Up axis" (assuming $\mathrm{Y}=$ up)
- How to update transformation $T_{D}$ ? Two ways:
- $T_{D} \leftarrow T_{D} \cdot T_{\text {new }}=$ object $D$ rotates around its up axis
(e.g. going supine to prone if laying down)
- $T_{D} \leftarrow T_{\text {new }} \cdot T_{D} \quad=$ object $D$ rotates in world's up axis


## Composite scenes:

## hierarchical transformations

- So far, we assumed that the transform of each object goes from local to global in one step
- In reality, the scene is defined hierarchically
- Objects have sub-objects in them
- a city is made of houses made of walls made of bricks
- a «hat» sits on an «head» which sits on a «character» which sits in a «spaceship» moving across the «scene»
- Also: different instances of the same object can appear in multiple locations of the scene
- E.g. all wheels of all cars are the same "wheel" model


18

## Scene graph

A tree (i.e. hierarchical structrure)

- Each nodes: a space (a reference frame)
- The Local Space of that node
- To each node we associate:
- Instances to... stuff:
anything at all that has a place in the virtual scene:
- 3D models, lights, cameras, virtual microphones spawn points, explosions, etc
- Root node: world space
- Global Space = local space of the root
- On the arches: we associate the transform - the "local" trasform



## Local VS global Transform

- Local transform (a.k.a. «relative» transform)
- from the local space of a node to the local space of its parent space
- Global transform (a.k.a. «absolute» transform)
- from the local space of a node to the world space (the "local" space of the root)
- obtained by: cumulating local transforms to the root!
- benefit: changing the transform associate to a node affects its entire subtree!



23

Reminder: inverse of a composite transform (or, in general, function)
global transform $T B \circ T_{A}$
of $A$


Inverse of global transform of $A$
$\longrightarrow$ : compute as needed

Reminder: inverse of a composite transform (or, in general, function)

$$
(T B \circ T A)^{-1}=T A^{-1} \circ T B^{-1}
$$

- The inverse of "first Ta then Tb " is
"the inverse of Tb" followed by "the inverse of Ta"
- As it's natural: if you...
- "take a step forward, then turn by $90^{\circ}$ on the left"
...then, to go back to the starting pos you need to...
- "turn by $90^{\circ}$ on the right, then take a step backward"


## The camera in the scene graph

- Camera:
- Like any other object in the scene, the camera sits in a node the scene-graph
- for the scene to be rendered, there must be a camera somewhere in the graph!
- View Space = Local Space of the camera
- (Screen Space is a similar, and sometimes equivalent, concept)
- the View Space is crucial for the rendering engine
- In view space, coordinates describe where things are in front of the camera!
- For example: $z>0 \Rightarrow$ in front of the camera, $z<0 \Rightarrow$ behind the camera (don't render)
- Camera animations = move camera
- by anything that changes its global transformation
- e.g. a script changing its local transform, or the one of it's parent


## The camera is in the scene graph

E.g.: to make the camera follow the car...


27

## Transforms for the Graphics engine (link to Computer Graphics courses)

- The rendering engine uses a few standard transformations, when rendering an object,
- They are named:
- "Model" matrix: from object space to world space
- Captures how the scene is modelled (by a scener)
- "View" matrix: from world space to view space
- Captures how the scene is viewed (by the camera)
- "Model-View" matrix: from object space to view space
- ( "matrix" because trasnforms are usually modelled as $4 \times 4$ matrices by Rendering engines \& APIs)
- Computing them from the scene graph is easy


29


## Authoring a 3D scene in a game

- E.g. as a part of the Level Design
- Two different parts, by different artists:

ใll 3D modellers make «scene props»

- the 3D models to beatistsembled
- (including their texutres etc)
sceners compose the scene
- they assemble the props into a Scene Graph


## Authoring a 3D scene

- Examples of other assets associated to a scene - a Collision Mesh (a Geometry Proxy)
- one for each "solid" scene-prop
- can this be made automatic? Possible, not easy
- assigned to nodes (for dynamic objects), or (for static objects) possibly all merged into one
- needed for: physics, visibility computation, AI, plus all sorts of gameplay reasons...
- a Navigation Mesh (aka Al mesh)
- usually, one for the entire scene (stored in the root node)
- needed by: AI (routing - see later)
- can this be made automatic? Possible, not tribial


## Authoring a 3D scene

- Examples of other assets associated to a scene:
- Scripts
- by the level designer
- Sky box
- Outer terrain mesh...
- Ambient
sounds
- Other data such as spawn points, and more


## Scene Graph as a data structure

- Each engine / library adopts its own solution
- No standards
- but file formats exists which can include a scene graph: e.g. COLLADA

Typical concepts:

- each Node class stores
- the local transform
- link to parent
- maybe, and/or to children, sibilings...)
- links to instances / assets
- global transforms / inverse are computed on demand
- some mechanism is used for repeated sub-trees


35


36


37

## Nodes of a scene-graph in Unity GameObjects \& Transforms

A node = a GameObject with

- a transform field, containing
- its local transform
- links to Parent, Children (and siblings) - which are transforms
- any number of associated "components", which represent anything residing in that node, like
- Meshes (to display at this nodes)
- Cameras: active one(s) produces the rendering(s)
- "RigidBodies": objects controlled by the physics
- "Colliders": geom proxies used for collisions
- "Particle systems" : (i.e. the "emitters" of particles)
- Sound producers / receivers
- Scripts ...
- basically any asset!


## Nodes of a scene-graph in

## GameObjects \& Transforms

- The Transformation actually stores the local transf:
- localPosition, localRotation, localScale
- goes from a node to its parent
- the Global transformation can be accessed via the properties:
- position, rotation, scale assigning / reading a field, actually means invoking ("global" is left implicit) setters/getters (C\# trick)
- what does getting / setting them really do? exercise!
- this it doesn't always work for "scale"! why?
(A: it's because anisotropy)
40


## Digression on \&unity: <br> properties and components

- Properties (C\# mechanism) it feels like a field (you can read or assign it) but it's actually a getter and setter method
- obj. $x x=3 \quad$...means... obj.set_xx( 3 )
- $\mathbf{f o o}=\mathrm{obj} . \mathrm{xx}$...means... foo $=$ obj.get_xx()
- Components (Unity library mechanism)
- A generic something attached to a GameObject
- GameObject g; g.getComponent< type >()
base class returns component of required type for everything (if it exists)


## Nodes of a scene-graph in (11) Unrean

 USceneComponentA node within a graph with:

- link to parent / children:
- getParentComponents
- getChildComponent( index )
- associated stuff to it:

UPrimitiveComponent (subclass)

- for models, physical bodies, etc
- Local Transform: (fields)
- RelativeLocation, RelativeRotation, RelativeScale
- Global Transform: (methods)
- GetComponentTransform() /* return transformation */


## Mechanisms for shared subtrees

- In Unity: see "Prefabs"
- In Unreal: see "BluePrints"


44

## Exercises 1/2

What is the new transform $\mathrm{T}^{\prime} 7$ which should subtitute T7 if...

- ...node Lil reattached as a child of $D$, leaving its position in world space unaffected (e.g. by a scener, or a script)
- ...node D is attached under node $L$, without affecting its world space position.
- ...the object in node L must be moved 1 unit on the right in view space (camera is in node C)
- ...the object in node L must be moved by 1 unit ON ITS RIGHT ...the object in node L must be displaced by a new transform $T$ applied in post-transform space.

Note: these kinds of problems are silently solved by Unity all the times (in the scripts \& when user manipulates the the GUI)

46

## Exercises 2/2

- Report the global transform of node L
- I place a camera in node H: report the View Transform for this scene
- What does it mean to apply a translation $(0,2,0)$ to $L$...

1. in L Space (the local space of L)?
2. in World space?
3. in View Space?

- Say T7 is the identity, and the camera is in H : how to modify T 7 to get the case 1,2 or 3 ?
- Find the origin of space $E$ in space $H$, and viceversa
- A microfone is in (the origin of) node E, and a speaker is in (the origin of) node H . Find the distance from the mic to the speaker

